Introduction to Propositional Logic

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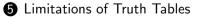
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1 PL: Truth Tables

- **2** Using Truth Tables to Determine the Truth Value of a Complex Wff
- **3** The Truth-Table Method
- Truth-Table Analysis
 Contradiction, Tautology, Contingency
 Consistency
 Equivalence
 Validity



Truth Tables: An Introduction

Imagination test: an argument can be identified as *deductively valid* if and only if it was impossible to imagine a scenario where all of the premises were true and conclusion false.

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Imagination test: an argument can be identified as *deductively valid* if and only if it was impossible to imagine a scenario where all of the premises were true and conclusion false.

- If you can imagine such a scenario, then the argument is invalid.
- If you cannot imagine such a scenario, then either the argument is valid.

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- People sometimes judge that certain arguments are "valid" because (1) they believe the conclusion and (2) are unwilling to consider scenarios where the premises are true and the conclusion is false.
- 3 Arguments about abstract topics can be difficult to imagine

A New Test

What we want then is a new test that

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- 1 does not rely upon the limited imaginative powers of human beings
- **2** is immune to bias
- **3** can be formulated about any subject matter.

Definition (truth table)

A truth table for PL is a table that provides a graphical way of representing valuations of wff(s) under a set of interpretations.

A truth table can be used to mechanically test sets of wffs and arguments.

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Definition (decision procedure)

A decision procedure is a mechanical method that determines in a finite number of steps whether a proposition, set of propositions, or argument has a certain logical property (one of these being whether or not an argument is deductively valid!).

Truth Tables: Step by Step

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- Step 3: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

Truth Table for Five Complex Wffs

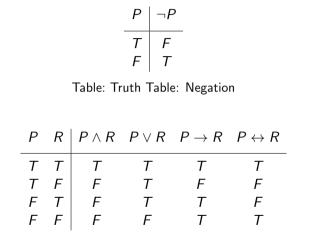


Table: Truth Table: Conjunction, Disjunction, Conditional, and Biconditional

Let's determine the truth value for $P \to \neg R$ under a single interpretation of P and R: $\mathscr{I}(P) = T$ and $\mathscr{I}(R) = F$. Let's determine the truth value for $P \to \neg R$ under a single interpretation of P and R: $\mathscr{I}(P) = T$ and $\mathscr{I}(R) = F$.

• Step 1: Write out the formula or set of formulas you want to test.

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• Step 1: Write out the formula or set of formulas you want to test.

$$P \rightarrow \neg R$$

Next, consider how $P \rightarrow \neg R$ is constructed.

- $\bullet P \text{ is a wff }$
- \mathbf{O} R is a wff
- **3** If R is a wff, then $\neg R$ is a wff

Next, consider how $P \rightarrow \neg R$ is constructed.

- $\bullet P \text{ is a wff }$
- $\mathbf{2} R$ is a wff
- **3** If R is a wff, then $\neg R$ is a wff
- 4 If P is a wff and $\neg R$ is a wff, then $P \rightarrow \neg R$ is a wff.

Assign truth values to the subformulas of $P \rightarrow \neg R$ in that order.

Start by writing truth values under the proposition letters. Since $\mathscr{I}(P) = T$ and $\mathscr{I}(R) = F$ write these values under the letters.

$$\begin{array}{ccc} P & \rightarrow & \neg & R \\ \hline T & & F \end{array}$$

P is a wff

- 2 R is a wff
- \bullet $\neg R$ is a wff
- $P \to \neg R \text{ is a wff.}$

The next wff constructed is $\neg R$ using R. So, determine the truth value of $\neg R$ using the truth value of R.

$$\begin{array}{ccc} P & \rightarrow & \neg & R \\ \hline T & & T & F \end{array}$$

P is a wff
2 R is a wff
$\Im \neg R$ is a wff

The next wff constructed is $P \rightarrow \neg R$ using P and $\neg R$. So, determine the truth value of $P \rightarrow \neg R$ using the truth value of P and $\neg R$.

$$\frac{P \rightarrow \neg R}{T T T F}$$

 $\bullet P is a wff$

- **2** *R* is a wff
- **3** $\neg R$ is a wff
- $P \to \neg R \text{ is a wff.}$

• We have seen how the truth value of a complex wff ϕ can be determined under a *single* interpretation of the propositional letters.

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- The truth-table method, however, is more general in that it allows for determining the truth value of wffs under all admissible interpretations of the propositional letters. For example, consider the following wff: ¬P ∨ ¬R.

Step 1: Write out the wff ϕ and all of the propositional letters in ϕ all of the propositional letters to the left of ϕ .

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Step 2: Write all possible interpretations for the propositional letters in ϕ under the propositional letters.

Truth-Table Method for n-Interpretations

Step 2: Write all possible interpretations for the propositional letters in ϕ under the propositional letters.

How many interpretations are there?

The number of possible interpretations (and therefore rows) is determined by the number of propositional letters in the set of wffs being considered. That is, the number of rows required for a truth table of any argument is determined by 2^n where *n* is the number of propositional letters in the argument.

- **()** Since P involves 1 propositional letter, $(2^1 = 2)$ rows are needed.
- **2** Since $P \wedge Q$ involves 2 propositional letters, $(2^2 = 4)$ rows are needed.
- **3** Since $(P \land Q) \land R$ involves 3 propositional letters, $(2^3 = 8)$ rows are needed.

Step 2: Write all possible interpretations for the propositional letters in ϕ under the propositional letters.

Ρ	R	_	Ρ	\vee	7	R
Т	Т					
Т	F					
F	Т					
F	F					

Step 3: For each row, write the truth values under the corresponding letter in the row.

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Ρ	R	-	Ρ	\vee	-	R
Т	Т		Т			Т
Т	F		Т			F
F	Т		F			Т
F	F		F			F

Step 4: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

Truth-Table Method for n-Interpretations

Step 4: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

		-			
Т	Т	F F T T	Т	F	Т
Т	F	F	Т	Т	F
F	Т	Т	F	F	Т
F	F	Т	F	Т	F

First determine the truth value of $\neg P$ using the truth value of P and the truth values of $\neg R$ using the truth value of R.

Step 4: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

The above shows the truth value of $\neg P \lor \neg R$ under all of the different ways that P and R can be interpreted in **PL**.

Truth-Table Analysis

So far, we see how truth tables can be used to:

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- ① determine the truth value of a complex wff under a single interpretation
- **2** determine the truth value of a complex wff under all interpretations

Truth-Table Analysis

Next, let's use truth tables to test whether:

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- () a wff ϕ is a contingency, tautology, or contradiction.
- **2** a pair of wffs ϕ , *psi* are equivalent
- 3 a collection of wffs ϕ,ψ are consistent
- ④ an "argument" is valid

Contradiction

Tautology

Contingency

Definition (Contradiction)

A proposition \mathbf{P} is a contradiction if and only if \mathbf{P} is always false.

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Definition (PL-Contradiction)

A wff ϕ is a PL-contradiction if and only if $v(\phi) = F$ under every interpretation.

Contingency, Tautology, Contradiction Tautology

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Definition (PL-Tautology)

A wff ϕ is a PL-tautology if and only if $v(\phi) = T$ under every interpretation.

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Definition (PL-Contingency)

A wff ϕ is a PL-contingency if and only if ϕ is neither a contradiction nor a tautology. Equivalently, ϕ is a PL-contingency if and only if ϕ is $v(\phi) = T$ under at least one intrepretation and $v(\phi) = F$ under at least one interpretation.

Contingency, Tautology, Contradiction How to Test

We can use a truth table to check whether a wff is a contradiction, tautology, or contingency by:

Contingency, Tautology, Contradiction How to Test

We can use a truth table to check whether a wff is a contradiction, tautology, or contingency by:

- Constructing the truth table
- Checking whether the wff is false under every interpretation (contradiction), true under every interpretation (tautology), or neither true nor false under every intrepretation (contingency).

Is $\neg P \lor \neg R$ a PL-contingency, PL-tautology, or PL-contradiction?

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It is a PL-contingency!

Is $P \lor \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

$$\begin{array}{c|cccc} P & P & \lor & \neg & P \\ \hline T & T & T & F & T \\ F & F & T & T & F \end{array}$$

Is $P \lor \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

$$\begin{array}{c|cccc} P & P & \lor & \neg & P \\ \hline T & T & T & F & T \\ F & F & T & T & F \end{array}$$

This wff is a tautology.

Is $P \land \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

$$\begin{array}{c|cccc} P & P & \wedge & \neg & P \\ \hline T & T & F & F & T \\ F & F & F & T & F \end{array}$$

Is $P \land \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

$$\begin{array}{c|cccc} P & P & \wedge & \neg & P \\ \hline T & T & F & F & T \\ F & F & F & T & F \end{array}$$

This wff is a contradiction.

It is not the case that if Liz does not eat ice cream, then she does not eat cake.

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- **2** Translate as $\neg(\neg I \rightarrow \neg C)$

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Ι	С	_	(_	Ι	\rightarrow	_	С)
Т	Т	F		F	Т	Т	F	Т	
Т	F	F		F	Т	Т	Т	F	
F	Т	Т		Т	F	F	F	Т	
F	F	F		Т	F	Т	Т	F	

The proposition "It is not the case that if Liz does not eat ice cream, then she does not eat cake" is a contingency since $\neg(\neg I \rightarrow \neg C)$ is a PL-contingency.

Consistency



Definition (Consistent)

A set of propositions is consistent iff they all can be true (there is some way that they all can be true).

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Definition (PL-Consistent)

A set of wffs Γ is PL-consistent if and only if there is at least one interpretation such that all of the members of Γ are true.

So, if ϕ and ψ are wffs, the set of wffs $\{\phi, \psi\}$ is PL-consistent provided there is at least one interpretation where both ϕ and ψ are true.

Definition (Inconsistent)

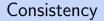
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Definition (Inconsistent)

A set of propositions is inconsistent iff they cannot all be true at the same time.

Definition (PL-Inconsistenct)

A set of wffs Γ is PL-inconsistent if and only if they are not PL-consistent (there is *no* there interpretation such that all of the members of Γ are true.)



Example (A Trivial Example)

- *P* and *Q* are PL-consistent.
- Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = T$

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- Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = T$
- If \$\mathcal{I}(P) = F\$ and \$\mathcal{I}(Q) = T\$, then there is an interpretation for which P and Q are both true.

Example (A Trivial Example)

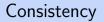
- P and Q are PL-consistent.
- Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = T$
- If $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = T$, then there is an interpretation for which P and Q are both true.
- Therefore, P and Q are PL-consistent.



In other cases, it isn't obvious.

Example (Example)

- Are $P \lor Q$ and $\neg (P \land Q)$ PL-consistent?
- What about $\neg P \lor Q$ and $\neg (P \land Q)$?
- What about $\neg P \lor (Q \lor \neg R)$ and $\neg (P \leftrightarrow Q)$?



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- Write each wff down in a row.
- **2** Construct a single truth table.
- S Check for whether there is at least one row where all the wffs in the row are T.
- If there is a row, then the test says the wffs are consistent.
- **⑤** If there is not a row, then the test says the wffs are inconsistent.

Let's test whether $P \rightarrow Q, P \land Q, P \lor \neg Q$ is PL-consistent.

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Notice that in the first row, all three wffs are true. Therefore, there is at least one interpretation where all the wffs are true. Therefore, the set is PL-consistent.

Let's test whether these wffs are PL-consistent: $(P \rightarrow Q), (\neg R \lor Q), R \land \neg Q$:

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Р	Q	R	(Ρ	\rightarrow	Q)	(R	V	Q)	(R	\wedge		Q)
Т	Т	Т		Т	Т	Т			F	Т	Т	Т			Т	F	F	Т	
Т	Т	F		Т	Т	Т			Т	F	Т	Т			F	F	F	Т	
Т	F	Т		Т	F	F			F	Т	F	F			Т	Т	Т	F	
Т	F	F		Т	F	F			Т	F	Т	F			F	F	Т	F	
F	Т	Т		F	Т	Т			F	Т	Т	Т			Т	F	F	Т	
F	Т	F		F	Т	Т			Т	F	Т	Т			F	F	F	Т	
F	F	Т		F	Т	F			F	Т	F	F			Т	Т	Т	F	
F	F	F		F	Т	F			Т	F	Т	F			F	F	Т	F	

Let's	test	t wh	ethe	er the	ese w	ffs a	are F	² L-c	consi	sten	t: (<i>F</i>	ho ightarrow	Q),	(¬/	$R \lor R$	Q), F	R ∧ -	$\neg Q$:	
Ρ	Q	R	(Ρ	\rightarrow	Q)	(R	V	Q)	(R	\wedge		Q)
Т	Т	Т		Т	Т	Т			F	Т	Т	Т			Т	F	F	Т	
Т	Т	F		Т	Т	Т			Т	F	Т	Т			F	F	F	Т	
Т	F	Т		Т	F	F			F	Т	F	F			Т	Т	Т	F	
Т	F	F		Т	F	F			Т	F	Т	F			F	F	Т	F	
F	Т	Т		F	Т	Т			F	Т	Т	Т			Т	F	F	Т	
F	Т	F		F	Т	Т			Т	F	Т	Т			F	F	F	Т	
F	F	Т		F	Т	F			F	Т	F	F			Т	Т	Т	F	
F	F	F		F	Т	F			Т	F	Т	F			F	F	Т	F	
Ther	re is	no r	ow y	wher	e all	the	wffs	are	еΤ, :	so tł	ie se	t is	PL-i	inco	nsist	tent.			

Example

• **Obvious:** "John is tall" and "Mary is tall" are consistent. They both can be true if "John is tall" is true and "Mary is tall" is true.

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- **Obvious:** "John is tall" and "Mary is tall" are consistent. They both can be true if "John is tall" is true and "Mary is tall" is true.
- Not So Obvious: "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."

 "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."

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- **2** Translate as $J \rightarrow M, \neg J \lor M$

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			\rightarrow					
T	Т	Т	T F T T	Т	F	Т	Т	Т
Т	F	T	F	F	F	Т	F	F
F	Т	F	Т	Т	T	F	Т	Т
F	F	F	Т	F	T	F	Т	F

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			\rightarrow					
Т	Т	Т	T F T T	Т	F	Т	Т	Т
Т	F	T	F	F	F	Т	F	F
F	Т	F	Т	Т	T	F	Т	Т
F	F	F	Т	F	T	F	Т	F

There is at least one row where the wffs are true. So, $J \rightarrow M, \neg J \lor M$ is PL-consistent, so the sentences are consistent.

Equivalence

Definition (Equivalent)

A pair of propositions P and Q are equivalent provided whenever P is true, Q is true and whenever P is false, Q is false.

Definition (PL-Equivalent)

A members of a set of wffs Γ are PL-equivalent iff for every interpretation of the members in $\gamma_1, \ldots \gamma_n \in \Gamma$, $v(\gamma_1) = \ldots v(\gamma_n)$.

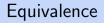
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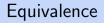
Definition (PL-Equivalent)

A members of a set of wffs Γ are PL-equivalent iff for every interpretation of the members in $\gamma_1, \ldots \gamma_n \in \Gamma$, $v(\gamma_1) = \ldots v(\gamma_n)$.

In other words, if you have a set of wffs ϕ , ψ and the truth values match for every interpretation, then the wffs are equivalent.



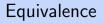
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- P and P are equivalent.
- If $\mathscr{I}(P) = T$, then $\mathscr{I}(P) = T$
- If $\mathscr{I}(P) = F$, then $\mathscr{I}(P) = F$

- *P* and *P* are equivalent.
- If $\mathscr{I}(P) = T$, then $\mathscr{I}(P) = T$
- If $\mathscr{I}(P) = F$, then $\mathscr{I}(P) = F$
- Thus, for every interpretation of *P* and *P*, whenever *P* is true, *P* is true, and whenever *P* is false, *P* is false.



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- Write each wff down in a row.
- **2** Construct a single truth table.
- **③** For each row, check for whether the truth values match
- If they match for each row, then the wffs are equivalent.
- **6** If they do not match for each row, then the wffs are not equivalent.

Equivalence

Are $P \leftrightarrow Q, P \rightarrow Q$ PL-equivalent?

						\rightarrow	
Т	Т	Т	Т	Т	Т	T F T T	Т
Т	F	Т	F	F	Т	F	F
F	Т	F	F	Т	F	Т	Т
F	F	F	Т	F	F	Т	F

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Ρ			\leftrightarrow				
Т	Т	Т	T F F T	Т	Т	Т	Т
Т	F	Т	F	F	T	F	F
F	Т	F	F	Т	F	Т	Т
F	F	F	Т	F	F	Т	F

No, they are not PL-equivalent.



 "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."



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Equivalence Application

- If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
- **2** Translate as $J \rightarrow M, \neg J \lor M$
- $\textbf{3 Create the table for } J \to M, \neg J \lor M$

Equivalence Application

- "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
- **2** Translate as $J \rightarrow M, \neg J \lor M$
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- **3** Create the table for $J \rightarrow M, \neg J \lor M$
- Check whether the wffs are equivalent or not equivalent.

			\rightarrow					
T	Т	Т	T F T T	Т	F	Т	Т	Т
Т	F	T	F	F	F	Т	F	F
F	Т	F	Т	Т	T	F	Т	Т
F	F	F	Т	F	T	F	Т	F

- "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
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Т	F	T	F	F	F	Т	F	F
F	Т	F	Т	Т	T	F	Т	Т
F	F	F	Т	F	T	F	Т	F

The wffs are PL-equivalent.

Validity



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If A, B, C are the premises and D is the conclusion of an argument, A, B, C semantically entail D iff there is no interpretation where A, B, C are all true and D is false.

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- We make use of the double turnstile "⊨" to express entailment ("models" or "semantically entails")
- **3** $\Gamma \models \psi$ says " Γ semantically entails ψ "
- **4** If it is not the case that Γ semantically entails ψ , then we write $\Gamma \not\models \psi$.

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- Write each wff down in a row.
- **2** Construct a single truth table.
- **③** For each row, check for a row where all the members of Γ are T and ψ is F.
- ④ If there is a row, then it is not the case that Γ semantically entails ψ . So, we write $\Gamma \not\models \psi$
- **6** If there is no such row, then it is the case that Γ semantically entails $\psi.$ So, we write $\Gamma\models\psi$

Does $P \rightarrow Q$ and P semantically entail Q?

			\rightarrow			Q
Т	Т	Т	T F T T	Т	Τ	Τ
Т	F	Т	F	F	Τ	F
F	Т	F	Т	Т	F	Т
F	F	F	Т	F	F	F

Does $P \rightarrow Q$ and P semantically entail Q?

Yes. Notice that there is no row where $P \to Q$ and P are true and Q is false. So, $P \to Q, P \models Q$

Semantic Entailment An Example

Does $P \lor Q$ and P semantically entail Q?

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No. Notice that in row (2) $P \lor Q$ and P are true and Q is false. So, $P \lor Q, P \not\models Q$

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Semantic Entailment Application

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			\rightarrow					J
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Т	F	T	F	F	Τ	F	F	Т
F	Т	F	Т	Т	F	Т	Т	F
F	F	F	Т	F	Τ	F	Т	F

Semantic Entailment Application

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- **2** Translate as $J \rightarrow M, \neg M \models \neg J$
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			\rightarrow				-	J
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Т	F	T	F	F	Τ	F	F	Т
F	Т	F	Т	Т	F	Т	Т	F
F	F	F	T	F	Τ	F	Т	F

 $J \rightarrow M, \neg M$ semantically entails $\neg J$

- **1** If ψ is a PL-tautology, then $\Gamma \models \psi$.
- **2** Since $\Gamma \not\models \psi$ only when ψ is false, check rows where ψ is false.
- **3** If Γ is PL-inconsistent, then $\Gamma \models \psi$.
- **③** Since $\Gamma \not\models \psi$ only when Γ is a PL-consistent, check rows where all the wffs in Γ are truie.

• Problem 1: Provided an argument is capable of being fully expressed by a truth-functional language like **PL**, the truth-table method seemingly guarantees there is a way to determine whether that argument is "valid" or "invalid", but not every English argument can be represented in a truth-functional language like PL. There are some arguments in English that are valid, but are not valid in PL.

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- Problem 2: The truth-table test's complexity increases exponentially. For every new propositional letter introduced, the table grows:
 21 2, 22 4, 23 8, 24 16, 25 22, 26 64, 27 128

$$2^{1} = 2, 2^{2} = 4, 2^{3} = 8, 2^{4} = 16, 2^{5} = 32, 2^{6} = 64, 2^{7} = 128$$