Introduction to Propositional Logic

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Imagination test: an argument can be identified as *deductively valid* if and only if it was impossible to imagine a scenario where all of the premises were true and conclusion false.

- If you can imagine such a scenario, then the argument is invalid.
- If you cannot imagine such a scenario, then either the argument is valid.

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2. People sometimes judge that certain arguments are "valid" because (1) they believe the conclusion and (2) are unwilling to consider scenarios where the premises are true and the conclusion is false.
3. Arguments about abstract topics can be difficult to imagine

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1. does not rely upon the limited imaginative powers of human beings
2. is immune to bias
3. can be formulated about any subject matter.

Definition (truth table)

A truth table for **PL** is a table that provides a graphical way of representing valuations of wff(s) under a set of interpretations.

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Definition (decision procedure)

A decision procedure is a mechanical method that determines in a finite number of steps whether a proposition, set of propositions, or argument has a certain logical property (one of these being whether or not an argument is deductively valid!).

- Step 1: Write down the wff.

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- Step 2: Write the truth value (T or F) under each propositional letter in the wff for each interpretation $\mathscr{I}$.

Truth Tables: Step by Step

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- Step 2: Write the truth value (T or F) under each propositional letter in the wff for each interpretation $\mathscr{I}$.
- Step 3: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

P	$\neg P$
T	F
F	T

Table: Truth Table: Negation

P	R	$P \wedge R$	$P \vee R$	$P \rightarrow R$	$P \leftrightarrow R$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Table: Truth Table: Conjunction, Disjunction, Conditional, and Biconditional

Truth Table: An Example

Let's determine the truth value for $P \to \neg R$ under a single interpretation of P and R: $\mathscr{I}(P) = T$ and $\mathscr{I}(R) = F$.

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- Step 1: Write out the formula or set of formulas you want to test.

P	$\to$	$\neg$	R

Next, consider how $P \to \neg R$ is constructed.

1. P is a wff
2. R is a wff
3. If R is a wff, then $\neg R$ is a wff
4. If P is a wff and $\neg R$ is a wff, then $P \to \neg R$ is a wff.

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1. P is a wff
2. R is a wff
3. If R is a wff, then $\neg R$ is a wff
4. If P is a wff and $\neg R$ is a wff, then $P \to \neg R$ is a wff.

Assign truth values to the subformulas of $P \to \neg R$ in that order.

Start by writing truth values under the proposition letters. Since $\mathscr{I}(P) = T$ and $\mathscr{I}(R) = F$ write these values under the letters.

$$\begin{array}{cccc} P & \rightarrow & \neg & R \\ \hline T & & & F \end{array}$$

1. P is a wff
2. R is a wff
3. $\neg R$ is a wff
4. $P \rightarrow \neg R$ is a wff.

The next wff constructed is $\neg R$ using R. So, determine the truth value of $\neg R$ using the truth value of R.

P	$\rightarrow$	$\neg$	R
T		T	F

1. P is a wff
2. R is a wff
3. $\neg R$ is a wff
4. $P \rightarrow \neg R$ is a wff.

The next wff constructed is $P \to \neg R$ using P and $\neg R$. So, determine the truth value of $P \to \neg R$ using the truth value of P and $\neg R$.

P	$\to$	$\neg$	R
T	T	T	F

1. P is a wff
2. R is a wff
3. $\neg R$ is a wff
4. $P \to \neg R$ is a wff.

- We have seen how the truth value of a complex wff ϕ can be determined under a *single* interpretation of the propositional letters.

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- The truth-table method, however, is **more general** in that it allows for determining the truth value of wffs under **all** admissible interpretations of the propositional letters. For example, consider the following wff: $\neg P \vee \neg R$.

Step 1: Write out the wff ϕ and all of the propositional letters in ϕ all of the propositional letters to the left of ϕ.

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P	R	$\neg$	P	$\vee$	$\neg$	R

Step 2: Write all possible interpretations for the propositional letters in ϕ under the propositional letters.

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How many interpretations are there?

The number of possible interpretations (and therefore rows) is determined by the number of propositional letters in the set of wffs being considered. That is, the number of rows required for a truth table of any argument is determined by 2^n where n is the number of propositional letters in the argument.

1. Since P involves 1 propositional letter, $(2^1 = 2)$ rows are needed.
2. Since $P \wedge Q$ involves 2 propositional letters, $(2^2 = 4)$ rows are needed.
3. Since $(P \wedge Q) \wedge R$ involves 3 propositional letters, $(2^3 = 8)$ rows are needed.

Step 2: Write all possible interpretations for the propositional letters in ϕ under the propositional letters.

P	R	$\neg$	P	$\vee$	$\neg$	R
T	T					
T	F					
F	T					
F	F					

Step 3: For each row, write the truth values under the corresponding letter in the row.

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P	R	$\neg$	P	$\vee$	$\neg$	R
T	T		T			T
T	F		T			F
F	T		F			T
F	F		F			F

Step 4: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

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P	R	$\neg$	P	$\vee$	$\neg$	R
T	T	F	T		F	T
T	F	F	T		T	F
F	T	T	F		F	T
F	F	T	F		T	F

First determine the truth value of $\neg P$ using the truth value of P and the truth values of $\neg R$ using the truth value of R.

Step 4: Assign T or F to subformulas in the order that the wff is constructed using (1) the truth values assigned to the wffs used to construct those subformulas and (2) the valuation function associated with the operator introduced in the construction.

P	R	$\neg$	P	$\vee$	$\neg$	R
T	T	F	T	F	F	T
T	F	F	T	T	T	F
F	T	T	F	T	F	T
F	F	T	F	T	T	F

The above shows the truth value of $\neg P \vee \neg R$ under all of the different ways that P and R can be interpreted in **PL**.

So far, we see how truth tables can be used to:

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1. determine the truth value of a complex wff under a single interpretation
2. determine the truth value of a complex wff under all interpretations

Next, let's use truth tables to test whether:

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1. a wff ϕ is a contingency, tautology, or contradiction.
2. a pair of wffs ϕ, *psi* are equivalent
3. a collection of wffs ϕ, ψ are consistent
4. an "argument" is valid

Contradiction
Tautology
Contingency

Definition (Contradiction)

A proposition **P** is a contradiction if and only if **P** is always false.

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Definition (PL-Contradiction)

A wff ϕ is a PL-contradiction if and only if $v(\phi) = F$ under every interpretation.

Definition (Tautology)

A proposition **P** is a tautology if and only if **P** is always true.

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Definition (PL-Tautology)

A wff ϕ is a PL-tautology if and only if $v(\phi) = T$ under every interpretation.

Definition (Contingency)

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Definition (PL-Contingency)

A wff ϕ is a PL-contingency if and only if ϕ is neither a contradiction nor a tautology. Equivalently, ϕ is a PL-contingency if and only if ϕ is $v(\phi) = T$ under at least one intrepretation and $v(\phi) = F$ under at least one interpretation.

We can use a truth table to check whether a wff is a contradiction, tautology, or contingency by:

We can use a truth table to check whether a wff is a contradiction, tautology, or contingency by:

1. Constructing the truth table
2. Checking whether the wff is false under every interpretation (contradiction), true under every interpretation (tautology), or neither true nor false under every intrepretation (contingency).

Is $\neg P \vee \neg R$ a PL-contingency, PL-tautology, or PL-contradiction?

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P	R	$\neg$	P	$\vee$	$\neg$	R
T	T	F	T	F	F	T
T	F	F	T	T	T	F
F	T	T	F	T	F	T
F	F	T	F	T	T	F

Is $\neg P \vee \neg R$ a PL-contingency, PL-tautology, or PL-contradiction?

P	R	$\neg$	P	$\vee$	$\neg$	R
T	T	F	T	F	F	T
T	F	F	T	T	T	F
F	T	T	F	T	F	T
F	F	T	F	T	T	F

It is a PL-contingency!

Is $P \vee \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

P	P	$\vee$	$\neg$	P
T	T	T	F	T
F	F	T	T	F

Is $P \vee \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

P	P	$\vee$	$\neg$	P
T	T	T	F	T
F	F	T	T	F

This wff is a tautology.

Is $P \wedge \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

P	P	$\wedge$	$\neg$	P
T	T	F	F	T
F	F	F	T	F

Is $P \wedge \neg P$ a PL-contingency, PL-tautology, or PL-contradiction?

P	P	$\wedge$	$\neg$	P
T	T	F	F	T
F	F	F	T	F

This wff is a contradiction.

1. It is not the case that if Liz does not eat ice cream, then she does not eat cake.

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2. Translate as $\neg(\neg I \rightarrow \neg C)$

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2. Translate as $\neg(\neg I \to \neg C)$
3. Create the table for $\neg(\neg I \to \neg C)$

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2. Translate as $\neg(\neg I \rightarrow \neg C)$
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1. It is not the case that if Liz does not eat ice cream, then she does not eat cake.
2. Translate as $\neg(\neg I \rightarrow \neg C)$
3. Create the table for $\neg(\neg I \rightarrow \neg C)$
4. Check whether the wff is a PL-contingency, PL-tautology, or PL-contradiction.

I	C	$\neg$	(	$\neg$	I	$\rightarrow$	$\neg$	C	)
T	T	F		F	T	T	F	T	
T	F	F		F	T	T	T	F	
F	T	T		T	F	F	F	T	
F	F	F		T	F	T	T	F	

The proposition "It is not the case that if Liz does not eat ice cream, then she does not eat cake" is a contingency since $\neg(\neg I \rightarrow \neg C)$ is a PL-contingency.

Consistency

Definition (Consistent)

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A set of propositions is consistent iff they all can be true (there is some way that they all can be true).

Definition (PL-Consistent)

A set of wffs Γ is PL-consistent if and only if there is at least one interpretation such that all of the members of Γ are true.

So, if ϕ and ψ are wffs, the set of wffs $\{\phi, \psi\}$ is PL-consistent provided there is at least one interpretation where both ϕ and ψ are true.

Definition (Inconsistent)

A set of propositions is inconsistent iff they cannot all be true at the same time.

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A set of propositions is inconsistent iff they cannot all be true at the same time.

Definition (PL-Inconsistenct)

A set of wffs Γ is PL-inconsistent if and only if they are not PL-consistent (there is *no* there interpretation such that all of the members of Γ are true.)

In many cases we can just see that a set of wffs is PL-consistent.

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Example (A Trivial Example)

- P and Q are PL-consistent.
- Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = T$

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Example (A Trivial Example)

- P and Q are PL-consistent.
- Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = T$
- If $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = T$, then there is an interpretation for which P and Q are both true.

In many cases we can just see that a set of wffs is PL-consistent.

Example (A Trivial Example)

- P and Q are PL-consistent.
- Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = T$
- If $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = T$, then there is an interpretation for which P and Q are both true.
- Therefore, P and Q are PL-consistent.

In other cases, it isn't obvious.

Example (Example)

- Are $P \vee Q$ and $\neg(P \wedge Q)$ PL-consistent?
- What about $\neg P \vee Q$ and $\neg(P \wedge Q)$?
- What about $\neg P \vee (Q \vee \neg R)$ and $\neg(P \leftrightarrow Q)$?

A truth table offers a method for testing whether a set of propositions is consistent or inconsistent.

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1. Write each wff down in a row.
2. Construct a single truth table.
3. Check for whether there is *at least one row* where all the wffs in the row are T.
4. If there is a row, then the test says the wffs are consistent.
5. If there is not a row, then the test says the wffs are inconsistent.

Let's test whether $P \to Q, P \wedge Q, P \vee \neg Q$ is PL-consistent.

P	Q	P	$\to$	Q	P	$\wedge$	Q	P	$\vee$	$\neg$	Q
T	T	T	T	T	T	T	T	T	T	F	T
T	F	T	F	F	T	F	F	T	T	T	F
F	T	F	T	T	F	F	T	F	F	F	T
F	F	F	T	F	F	F	F	F	T	T	F

Let's test whether $P \to Q, P \land Q, P \lor \neg Q$ is PL-consistent.

P	Q	P	$\to$	Q	P	$\land$	Q	P	$\lor$	$\neg$	Q
T	T	T	T	T	T	T	T	T	T	F	T
T	F	T	F	F	T	F	F	T	T	T	F
F	T	F	T	T	F	F	T	F	F	F	T
F	F	F	T	F	F	F	F	F	T	T	F

Notice that in the first row, all three wffs are true. Therefore, there is at least one interpretation where all the wffs are true. Therefore, the set is PL-consistent.

Let's test whether these wffs are PL-consistent: $(P \to Q), (\neg R \lor Q), R \land \neg Q$:

Let's test whether these wffs are PL-consistent: $(P \to Q), (\neg R \vee Q), R \wedge \neg Q$:

P	Q	R	(	P	$\to$	Q	)	(	$\neg$	R	$\vee$	Q	)	(	R	$\wedge$	$\neg$	Q	)
T	T	T		T	T	T			F	T	T	T			T	F	F	T	
T	T	F		T	T	T			T	F	T	T			F	F	F	T	
T	F	T		T	F	F			F	T	F	F			T	T	T	F	
T	F	F		T	F	F			T	F	T	F			F	F	T	F	
F	T	T		F	T	T			F	T	T	T			T	F	F	T	
F	T	F		F	T	T			T	F	T	T			F	F	F	T	
F	F	T		F	T	F			F	T	F	F			T	T	T	F	
F	F	F		F	T	F			T	F	T	F			F	F	T	F	

Let's test whether these wffs are PL-consistent: $(P \to Q), (\neg R \vee Q), R \wedge \neg Q$:

P	Q	R	(	P	$\to$	Q	)	(	$\neg$	R	$\vee$	Q	)	(	R	$\wedge$	$\neg$	Q	)
T	T	T		T	T	T			*F*	T	T	T			T	*F*	*F*	T	
T	T	*F*		T	T	T			T	*F*	T	T			*F*	*F*	*F*	T	
T	*F*	T		T	*F*	*F*			*F*	T	*F*	*F*			T	T	T	*F*	
T	*F*	*F*		T	*F*	*F*			T	*F*	T	*F*			*F*	*F*	T	*F*	
F	T	T		*F*	T	T			*F*	T	T	T			T	*F*	*F*	T	
F	T	*F*		*F*	T	T			T	*F*	T	T			*F*	*F*	*F*	T	
F	*F*	T		*F*	T	*F*			*F*	T	*F*	*F*			T	T	T	*F*	
F	*F*	*F*		*F*	T	*F*			T	*F*	T	*F*			*F*	*F*	T	*F*	

There is no row where all the wffs are T, so the set is PL-inconsistent.

Example

- **Obvious:** "John is tall" and "Mary is tall" are consistent. They both can be true if "John is tall" is true and "Mary is tall" is true.

Example

- **Obvious:** "John is tall" and "Mary is tall" are consistent. They both can be true if "John is tall" is true and "Mary is tall" is true.
- **Not So Obvious:** "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."

1. "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."

1. "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
2. Translate as $J \rightarrow M, \neg J \vee M$

1. “If John is tall, then Mary is happy” and “John is not tall or Mary is happy.”
2. Translate as $J \to M, \neg J \vee M$
3. Create the table for $J \to M, \neg J \vee M$

1. "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
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4. Check whether the wff is a PL-consistent or PL-inconsistent.

1. "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
2. Translate as $J \rightarrow M, \neg J \vee M$
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4. Check whether the wff is a PL-consistent or PL-inconsistent.

J	M	J	$\rightarrow$	M	$\neg$	J	$\vee$	M
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

1. "If John is tall, then Mary is happy" and "John is not tall or Mary is happy."
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4. Check whether the wff is a PL-consistent or PL-inconsistent.

J	M	J	$\rightarrow$	M	$\neg$	J	$\vee$	M
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

There is at least one row where the wffs are true. So, $J \rightarrow M, \neg J \vee M$ is PL-consistent, so the sentences are consistent.

Equivalence

Definition (Equivalent)

A pair of propositions P and Q are equivalent provided whenever P is true, Q is true *and* whenever P is false, Q is false.

Definition (PL-Equivalent)

A members of a set of wffs Γ are PL-equivalent iff for every interpretation of the members in $\gamma_1, \ldots \gamma_n \in \Gamma$, $v(\gamma_1) = \ldots v(\gamma_n)$.

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A pair of propositions P and Q are equivalent provided whenever P is true, Q is true *and* whenever P is false, Q is false.

Definition (PL-Equivalent)

A members of a set of wffs Γ are PL-equivalent iff for every interpretation of the members in $\gamma_1, \ldots \gamma_n \in \Gamma$, $v(\gamma_1) = \ldots v(\gamma_n)$.

In other words, if you have a set of wffs ϕ, ψ and the truth values match for every interpretation, then the wffs are equivalent.

Example (A Trivial Example)

- P and P are equivalent.

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- P and P are equivalent.
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- If $\mathscr{I}(P) = F$, then $\mathscr{I}(P) = F$

Example (A Trivial Example)

- P and P are equivalent.
- If $\mathscr{I}(P) = T$, then $\mathscr{I}(P) = T$
- If $\mathscr{I}(P) = F$, then $\mathscr{I}(P) = F$
- Thus, for every interpretation of P and P, whenever P is true, P is true, and whenever P is false, P is false.

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A truth table offers a method for testing whether the members in a set of wffs are equivalent.

1. Write each wff down in a row.
2. Construct a single truth table.
3. For each row, check for whether the truth values match
4. If they match for each row, then the wffs are equivalent.
5. If they do not match for each row, then the wffs are not equivalent.

Are $P \leftrightarrow Q$, $P \rightarrow Q$ PL-equivalent?

P	Q	P	$\leftrightarrow$	Q	P	$\rightarrow$	Q
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
F	T	F	F	T	F	T	T
F	F	F	T	F	F	T	F

Are $P \leftrightarrow Q$, $P \to Q$ PL-equivalent?

P	Q	P	$\leftrightarrow$	Q	P	$\to$	Q
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
F	T	F	F	T	F	T	T
F	F	F	T	F	F	T	F

No, they are not PL-equivalent.

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P	Q	J	$\to$	M	$\neg$	J	$\vee$	M
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
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The wffs are PL-equivalent.

Validity

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If A, B, C are the premises and D is the conclusion of an argument, A, B, C semantically entail D iff there is no interpretation where A, B, C are all true and D is false.

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2. We make use of the double turnstile "$\models$" to express entailment ("models" or "semantically entails")
3. $\Gamma \models \psi$ says "Γ semantically entails ψ"
4. If it is not the case that Γ semantically entails ψ, then we write $\Gamma \not\models \psi$.

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1. Write each wff down in a row.
2. Construct a single truth table.
3. For each row, check for a row where all the members of Γ are T and ψ is F.
4. If there is a row, then it is not the case that Γ semantically entails ψ. So, we write $\Gamma \not\models \psi$
5. If there is no such row, then it is the case that Γ semantically entails ψ. So, we write $\Gamma \models \psi$

Does $P \to Q$ and P semantically entail Q?

P	Q	P	$\to$	Q	P	$\models$	Q
T	T	T	T	T	T		T
T	F	T	F	F	T		F
F	T	F	T	T	F		T
F	F	F	T	F	F		F

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P	Q	P	$\to$	Q	P	$\models$	Q
T	T	T	T	T	T		T
T	F	T	F	F	T		F
F	T	F	T	T	F		T
F	F	F	T	F	F		F

Yes. Notice that there is no row where $P \to Q$ and P are true and Q is false. So, $P \to Q, P \models Q$

Does $P \vee Q$ and P semantically entail Q?

P	Q	P	$\vee$	Q	P	$\models$	Q
T	T	T	T	T	T		T
T	F	T	T	F	T		F
F	T	F	T	T	F		T
F	F	F	F	F	F		F

Does $P \vee Q$ and P semantically entail Q?

P	Q	P	$\vee$	Q	P	$\models$	Q
T	T	T	T	T	T		T
T	F	T	T	F	T		F
F	T	F	T	T	F		T
F	F	F	F	F	F		F

No. Notice that in row (2) $P \vee Q$ and P are true and Q is false. So, $P \vee Q, P \not\models Q$

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T	F	T	F	F	T	F		F	T
F	T	F	T	T	F	T		T	F
F	F	F	T	F	T	F		T	F

$J \rightarrow M, \neg M$ semantically entails $\neg J$

1. If ψ is a PL-tautology, then $\Gamma \models \psi$.
2. Since $\Gamma \not\models \psi$ only when ψ is false, check rows where ψ is false.
3. If Γ is PL-inconsistent, then $\Gamma \models \psi$.
4. Since $\Gamma \not\models \psi$ only when Γ is a PL-consistent, check rows where all the wffs in Γ are truie.

- Problem 1: Provided an argument is capable of being fully expressed by a truth-functional language like **PL**, the truth-table method seemingly guarantees there is a way to determine whether that argument is "valid" or "invalid", but not every English argument can be represented in a truth-functional language like PL. There are some arguments in English that are valid, but are not valid in PL.

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- Problem 2: The truth-table test's complexity increases exponentially. For every new propositional letter introduced, the table grows: $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^7 = 128$