Handout 6

RL: Symbols, Syntax, Semantics, Translation

6.1 Predicate Logic: Introduction

The language of **PL** has three principal strengths:

Strength 1 for any argument that is valid in **PL**, there is a corresponding valid argument in English. This means that some of the arguments that we wish to represent and the reasoning we do in English can be represented in the more precise language of **PL**.

Strength 2 There are decision procedures for **PL**! We can use tables and trees to mechanically check whether an argument is valid or invalid in **PL**, whether a set of propositions are consistent or inconsistent, etc.

Strength 3 There is a proof system (**PD**) for **PL**. That is, we have a codified set of rules that justify various derivations or moves forward in arguments.

The principal weakness of **PL** is the following:

Weakness 1 It is not expressive enough. That is, there are some valid arguments in English that cannot be represented in **PL**.

For example, consider the argument in Argument 6.1.

P1	All men are mortal.
P2	Socrates is a man.
C	Socrates is mortal.

Argument 6.1 – Argument for Socrates's mortality

To see why the above argument, when translated into **PL**, does not express a valid argument in **PL**, consider that when we translate a sentence form English to **PL**, we translate simple English sentences into single propositional letters. More complex wffs (e.g. $P \rightarrow Q$) express the logical structure between these propositional letters. In short, **PL** is a logic of complete sentences or propositions.

The validity of some arguments, however, depends upon the logical relationships between *parts* of sentences or propositions. For example, in the above example, the argument depends upon Socrates belongs to a class of men and how the class of men form a part of a more expansive class, namely mortals. In order to capture how parts of propositions relate to each other, we can devise a more expressive logical language, one that analyzes sentences at the sub-sentential level. This is the language of predicate logic (or the logic of relations): **RL**.

6.2 Symbols

First, we begin with the symbols of **RL**:

names	Lower case letters, a through v with or without numerical subscripts.
n-place predicates	Upper case letters, A through Z with or without numerical subscripts.
variables	Lower case letters, w through z with or without numerical subscripts.
operators	$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
parentheses	(,)
quantifiers	$\forall$, $\exists$

Table 6.1 – Symbols of **RL**

The above symbols form the vocabulary of **RL**. While, at this point, these symbols lack meaning, it is helpful to think of a non-formal analogue for each of these items.

1. **names** (a, b, c, d, e) are like proper names in that they refer to specific objects. For example, George Washington, Hurricane Sandy, Gandhi.
2. **n-place predicates** (A, B, C, D, E) are like predicate or relational terms that express properties or relations. For example, is blue, is happy, is taller than, etc.
3. **variables** are like placeholders for names of objects, e.g. in "x is happy" x is a placeholder for some name we could insert to make that statement true or false.
4. **quantifiers** are expressions for specifying the quantity of objects that have some property. For example, "Some", "All".

Exercise 1: *Determine what kind of symbol the following symbols are:*

1. a
2. c
3. g
4. z
5. x

6. $\rightarrow$
7. $\forall$
8. $\exists$

6.3 RL: Syntax

With the symbols of **RL** specified, we now turn to the syntax of **RL**. That is, the proper way of combining these symbols.

6.3.1 RL: Formation Rules

1. an n-place predicate P followed by n terms (names or variables) is a wff in **RL**.
2. If P is a wff in **RL**, then $\neg(P)$ is a wff in **RL**.[1]
3. If P and Q are wffs in **RL**, then $(P \wedge Q)$, $(P \vee Q)$, $(P \rightarrow Q)$, and $(P \leftrightarrow Q)$ are wffs in **RL**.
4. If P is a wff in **RL** containing a name a and if $P(x/a)$ is what results from substituting the variable x for every occurrence of a in P, then $(\forall x)P(x/a)$ and $(\exists x)P(x/a)$ are wffs in **RL** (provided that '$P(x/a)$' is not a wff).
5. Nothing else is a wff in **RL** except that which can be formed by repeated applications of the above.

In the first rule, reference is made to an "n-place predicate". Briefly, this is an uppercase letter with some number n of terms (variables or names) after it. More intuitively, English predicate and relational terms are the analogues of n-place predicate terms in that "is tall" is a one-place predicate in that there is one-place for a subject (or object) to go in order for "is tall" to express a complete sentence.

- "x is tall" is a 1-place predicate
- "x is taller than y" is a 2-place predicate
- "x is standing between y and z" is a 3-place predicate.

The above rules can be used to determine whether a formula is a wff. For an example, see Table 6.2.

1	Ra is a wff.	Rule (1)
2	Pa is a wff.	Rule (1)
3	If Ra is a wff, then $\neg(Ra)$ is a wff.	Line 1, rule (ii)
4	If Pa is a wff, then $\neg(Pa)$ is a wff.	Line 2, rule (ii)
5	If $\neg(Pa)$ is a wff in **RL** and $P(z/a)$ is what results from replacing as with zs, then $(\forall z)\neg(Pz)$ is a wff.	Line 4, Rule (iv)
6	If $\neg(Ra)$ is a wff and $(\forall z)\neg(Pz)$ is a wff, then $(\forall z)\neg(Pz) \wedge \neg Ra$ is a wff in **RL**.	Line 3, 5, rule (iii)

Table 6.2 – Example proving that $(\forall z)\neg Pz \wedge \neg Ra$ is a wff.

[1] We will make use of Conventions for parentheses. Thus $\neg(Pa)$ and $\neg Pa$ are both wffs.

Some examples will help illustrate which formulas are wffs and which are not wffs.

Example 1: Some examples of wffs and not wffs

wffs	not wffs
Pa	aP
Rab	aRb
Rxx	xRx
$\neg Pa \wedge Rba$	$\neg PaRba$
$(\forall x)Px$	$(\forall x)Py$
$(\exists x)(\forall y)Rxy$	$\exists x)(\forall x)Rxx$

Exercise 2: *Using the formation rules, show that the following propositions are wffs in RL, where 'Pxy' is a two-place predicate while 'Rx' and 'Zx' are one-place predicates:*

1. $Ra \wedge Paa$
2. $Raa \to Pa$
3. $(\forall x)Pxx$
4. $(\exists x)Px$
5. $\neg(\exists y)Pyy$
6. $\neg(\forall x)Pxx \wedge (\exists x)Zx$

6.3.2 Scope

The scope of the quantifiers is similar to the scope of the truth-functional operator for negation. That is, it applies to the wff to its immediate right.

$$(\forall x)Px$$

Without the use of parentheses, ambiguity is possible. Consider the following formula:

$$(\forall x)Px \to Rx$$

Does the quantifier ($\forall$) apply to Px alone or does it apply to the complex $Px \to Rx$? Here scope indicators (open and closed parentheses, braces, and brackets) are used to indicate the scope of the quantifier. Thus, in the above example, the universal quantifier applies only to Px while in the following formula, it operates upon the conditional '$Px \to Rx$':

$$(\forall x)(Px \to Rx)$$

Quantifiers can have other quantified formula in their scope. For example, in the following formula, the existential quantifier has a universally quantified wff in its scope:

$$(\exists x)(\forall y)(Px \to Ry)$$

Exercise 3: *In the following wffs, determine whether the variable is in the scope of a quantifier. If it is, state which quantifier has it in its scope.*

1. $(\forall x)Px$
2. $(\exists x)Rxy$
3. $(\forall x)Px \rightarrow Rxy$
4. $(\forall x)(Px \rightarrow Rxy)$
5. $(\forall x)(\exists y)Rxy$

6.3.3 Free vs. Bound Variables

Quantifiers specify the quantity of specific variables. For example, in the above formula, '$(\exists x)$' quantifies 'xs' while '$(\forall y)$' quantifies 'ys'. We call a variable that is quantified by a quantifier that quantifies for that specific variable, a bound variable. A variable that is not bound is a free variable. We call a variable is in the scope of a quantifier, irrespective of whether the quantifier quantifies for it, a scoped variable.

Definition – Bound Variable

A variable is bound if and only if it is in the scope of a quantifier that quantifies for that variable.

Definition – Free Variable

A variable is *free* if and only if it is not bound.

Example 1: Consider the following examples

1. $(\forall x)Fx$, x is bound
2. $(\forall x)Rxy$, x is bound, y is free even though it is in in the scope of $\forall$
3. $(\exists x)(Rx \wedge Lx)$, both xs are bound

Definition – Open formula

An open formula is a wff consisting of an n-place predicate P followed by n terms where one of those terms is a free variable

Definition – Closed formula

A closed formula is a wff consisting of an n-place predicate P followed by n terms where every term is a name or a bound variable.

Exercise 4: *Determine whether the formulas below are (i) wffs or not wffs and (ii) whether the formula is open or closed.*

1. Paa
2. $Paa \to La$
3. $(\forall x)Pxx$
4. $(\forall x)Pxy$
5. $(\exists x)(\forall y)(\forall z)(Pxy \to Lz)$
6. $(\exists x)(\forall y)(\forall z)Pxy \to Lz$

6.4 Semantics

In the semantics of **PL**, single propositional letters are assigned truth values (T or F) by an interpretation function while wffs in **PL** are assigned truth values by a valuation function. The semantics of **RL** is more complex in that the elementary symbols of **RL** better express parts of propositions or sentences rather than complete propositions.

A full account of the semantics of **RL** is outside of the scope of this course. It would require an excursion into some aspects of set theory, and this is more than we have time for in this class (if you are interested, email me).

Instead, we will look at the basics. To do this we need three key notions:

1. the domain of discourse
2. an interpretation function
3. a valuation function

The domain of discourse is all of the things we can talk about. You can think about it as a collection of objects in the world. Often the domain of discourse is narrower than *all* of the objects in the world. It will just include a certain selection of them. We will symbolize the domain of discourse with $\mathcal{D}$ and when we want to specify what objects are in $\mathcal{D}$, we will write $\mathcal{D}:$ and then write the objects we wish to talk about either by indicating a property it has or listing all of the objects

- $\mathcal{D}$: human beings
- $\mathcal{D}$: David, Liz, Tek, Ryan
- $\mathcal{D} : a, b, c, d$

Definition – Interpretation of RL

An interpretation of **RL** is a function that

1. assigns objects in $\mathcal{D}$ to each name in **RL**, and
2. assigns a set or collection of objects in $\mathcal{D}$ to n-place predicate terms.

In essence, it gives meaning to the names in **RL** by assigning each name an object in $\mathcal{D}$ and meaning to predicate terms by assigning each predicate term a set of objects.

We will symbolize the interpretation function as $\mathscr{I}$. For example, if a is a name in **RL**, an interpretation function would assign it a single object in $\mathcal{D}$ as follows: $\mathscr{I}(a) = a$. What this says is the name "a" is assigned the object a in the domain of discourse. More naturally, the name "David" refers to the object *David* in the world.

If R is a one-place predicate term, an interpretation function would assign it a set of objects in $\mathcal{D}$ as follows: $\mathscr{I}(R) := \{a, b, c, d\}$. What this says is the predicate term "R" is assigned a group of objects a, b, c, d from the domain. More naturally, the meaning of "x is red" or "red" is just the red things it refers to in the domain.

When doing with two, or three, or n-place predicate terms, we can make clear that we are referring to a pair of objects, or triplet, or n-tuple by using angle brackets. That is, to express the interpretation of "x is taller than y", we might write the following: $\mathscr{I}(T) := \{\langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$. What this says is that the "taller than" two-place predicate is interpreted in terms of pairs of things where the first object in the pair is taller than the other.

With the interpretation function in place, a valuation function can be used to determine the truth value of various wffs. Here we formulate a list of "valuation" rules:

Definition – Valuation in RL

Relative to a domain $\mathcal{D}$ and interpretation $\mathscr{I}$, a valuation (v or V) of a wff in **RL** is a function that assigns one and only one truth value (T or F) to each closed wff in **RL** in such a way (let a be any number of names, R be any n-place predicate, and P be any wff in **RL**)

1. $v(Ra_i) = T$ if and only if (iff) the interpretation of a_i is in R. That is, $\mathscr{I}(a_i) \in R$.
2. $v\neg(P) = T$ iff $v(P) = F$
3. $v(P \wedge Q) = T$ iff $v(P) = T$ and $v(Q) = T$
4. $v(P \vee Q) = T$ iff either $v(P) = T$ or $v(Q) = T$
5. $v(P \to Q) = T$ iff either $v(P) = F$ or $v(Q) = T$
6. $v(P \leftrightarrow Q) = T$ iff either $v(P) = T$ and $v(Q) = T$, or $v(P) = F$ and $v(Q) = F$
7. $v(\forall x)P = T$ iff for every name **a** not in P, every **a**-variant interpretation is such that $P(a/x) = T$.
8. $v(\exists x)P = T$ iff for at least one name **a** not in P, at least one **a**-variant interpretation is such that $P(a/x) = T$.

Exercise 5: *Let* $\mathcal{D} = \{a, b, c\}, \mathscr{I}(a) = a, \mathscr{I}(b) = b, \mathscr{I}(c) = c, \mathscr{I}(Hx) = \{a, b, c\}, \mathscr{I}(Lxy) = \{\langle a, a\rangle, \langle b, c\rangle\}$

1. Ha

2. $(\exists x)Hx$
3. $(\forall x)Hx$
4. $\neg(\forall x)Hx$
5. $(Ha \wedge Hb) \wedge Hc$
6. Lab
7. $\neg Laa$
8. Lba
9. $(\exists x)Lxx$
10. $(\forall x)Lxx$
11. $(\exists x)Lxx \vee (\exists x)(\exists y)Lxy$

6.5 RL: Translation

The formal language of **RL** can be used to express a fragment of the English language. The first step to any translation is to construct a translation key. A translation key does three things:

1. it stipulates the domain of discourse,
2. it interprets all names
3. it interprets all n-place predicates.

Here is an example of a translation key:

- $\mathcal{D}$: Living human beings
- a: Annie
- j: Jon
- f: Frank
- Txy: x is taller than y.
- Hx : x is happy.

Using the above translation key various English sentences can be translated into **RL** and closed formula in **RL** can be translated into English sentences. In **RL**, the n-place predicate is placed before (to the left) of the name. In English, however, typically we utter the name first and then the predicate.

Example 2:

1. Ha: a is H. Annie is happy.
2. Taj: a is taller than b. Annie is taller than Jon.
3. Tjf: Jon is taller than Frank:
4. Tfj: Frank is taller than Jon:
5. $\neg(Tjf \wedge Tfj)$: It is not both the case that Jon is taller than Frank and Frank is taller than Jon

The quantifier '$(\forall x)$' can be used to translate pseudo-English expressions like 'for every x', 'for all xs', 'for each x'. Likewise '$(\exists x)$' can be used to translate pseudo-English expressions like 'for some x', 'some xs', or 'there exists an x'.

Example 3:

1. $(\exists x)Hx$: Some x is H. At least one x is H. Someone is happy.
2. $(\forall x)Hx$: Every x is H. Everyone is happy.
3. $(\forall x)(\exists y)Txy$: For every x, there is some y, such that x is taller than y. Everyone is taller than someone.

The method for translating English expressions into quantified formula in **RL** is an art and so it can help to take a step-by-step method. One way to do this is to create a bridge translation between wffs in **RL** and sentences in English, and then use this bridge translation to translate into more colloquial English. A bridge translation is a half-way point between English and **RL**; it's not quite English and not quite **RL**: it's pseudo-English.

Consider the following translation key

- $\mathcal{D}$: human beings (living or dead)
- Hx: x is happy
- Zx: x is a zombie
- Mx: x is mortal
- Rx: x is murderer
- Wx: x is wrong

Now consider the following predicate wffs:

1. $(\forall x)Hx$
2. $(\forall x)\neg Zx$
3. $(\forall x)(Zx \rightarrow Hx)$
4. $(\forall x)(Zx \rightarrow \neg Hx)$
5. $\neg(\forall x)(Zx \rightarrow Hx)$

Let's consider a translation of (1) by taking one part of the formula at a time. '$(\forall x)$' is translated as 'For every x', 'for all xs', 'for each x'. The second part of (1) says 'x is H' or 'x is happy'. Putting these two parts together we get a bridge translation.

(1B) For every x, x is happy.

Using this bridge translation, we can more easily translate (1) into colloquial English:

(1E) Everyone is happy.

Consider a bridge translation of (2):

(2B) For every x, x is not a zombie.

Using (2B) we can render (2) into something more natural. There are two options.

(2E) Everyone is a not a zombie.

(2E*) No one is a zombie.

(2E*) is the preferable option as (2E) is ambiguous between two different wffs. While (2E) can express (2E*), it more naturally expresses the wff: $\neg(\forall x)Zx$, equivalently $(\exists x)\neg Zx$. For consider (2E) as a response to the question: "is everyone a zombie?". Responding with (2E) would convey the idea that some item in the domain of discourse does not have the property of being a zombie, rather than the stronger claim that no item in the discourse has that property.

Consider a bridge translation of (3):

(3B) For every x, if x is a zombie, then x is happy.

In the case that (3B) does not make it obvious how to render it into English, then you can try to make (3B) more concrete by expanding the bridge translation as follows:

(3B+) Choose any object you please in the domain of discourse, if that object is a zombie, then it will be also be happy.

Rendered into standard English, (3B) and (3B*) reads:

(3E) Every zombie is happy.

Consider a bridge translation of (4):

(4B) For every x, if x is a zombie, then x is not happy.

An additional bridge translation is the following:

(4B*) Choose any object you please in the domain of discourse consisting of human beings (living or dead), if that object is a zombie, then it will not be happy.

In colloquial English this is the following:

(4E) No zombies are happy.

Notice that in the case of (5), which is (5) $\neg(\forall x)(Zx \rightarrow Hx)$, the main operator is negation. One way to translate this by first translating '$(\forall x)(Zx \rightarrow Hx)$':

Every zombie is happy.

Next, translate the negation into English by putting 'Not' in front of this expression. That is, (5) reads:

(5E) Not every zombie is happy.

Finally, consider universally quantified expressions not involving $\rightarrow$ as the main operator

(6) $(\forall x)(Zx \wedge Hx)$

(7) $(\forall x)(Zx \vee Hx)$

(8) $(\forall x)(Zx \leftrightarrow Hx)$

These are

(6E) Everyone is a happy zombie.

(7E) Everyone is either a zombie or happy.

(8E) Everyone is a zombie if and only if they are happy.

Our focus thus far has been on the use of the universal quantifier to translate RL formula into English. Next, we turn to RL formulas that involve the existential quantifier. Consider the following predicate wffs:

1. $(\exists x)Hx$
2. $(\exists x)\neg Zx$
3. $\neg(\exists x)Zx$
4. $(\exists x)(Zx \wedge Hx)$
5. $(\exists x)Zx \wedge (\exists x)Hx$

Let's consider a translation of (1) by taking one part of the formula at a time. $(\exists x)$ is translated as 'For some x', 'there exists an x', 'there is at least one x'. The second part of (1) says 'x is H' or 'x is happy'. Putting thse two parts together we get a bridge translation. Again, a bridge translation is not quite English and not quite predicate logic. Here is a bridge translation of (1)

(1B) For some x, x is happy.

(1) says that there is at least one in the object in the $\mathcal{D}$ that has the property of being happy. Using the bridge translation (1B), we can more easily translate (1) into colloquial English:

(1E) Someone is happy.

Consider (2). Again, we can use a bridge translation,

(2B) For some x, x is not a zombie.

(2B) can be translated into colloquial English as follows:

(2E) Someone is not a zombie.

In the case of (3), note that negation has wide scope. Thus, we can translate '$(\exists x)Zx$' first, and then translate '$\neg(\exists x)Zx$'. That is, $(\exists x)Zx$ translates into 'Someone is a zombie', and $\neg(\exists x)Zx$ translates as:

(3E) It is not the case that someone is a zombie.

Notice that (2) and (3) say something distinct. (2) says that something exists that is not a zombie, while all (3) says is that zombies do not exist. Let's consider (4) and (5) together. The bridge translations for (4) and (5) are as follows:

(4B) For some x, x is a zombie and x is happy.

(5B) For some x, x is a zombie, and for some x, x is happy.

Notice that these two propositions do not say the same thing. (4) asserts that there is something that is both a zombie and happy, while (5) asserts that there is a zombie and there is someone who is happy.

Finally, consider some propositions where $\wedge$ is not the main operator.

(6) $(\exists x)(Zx \rightarrow Hx)$

(7) $(\exists x)(Zx \vee Hx)$

(8) $(\exists x)(Zx \leftrightarrow Hx)$

The bridge translations for these are

(6B) For some x, if x is a zombie, then x is happy.

(7B) For some x, x is a zombie or x is happy.

(8B) For some x, x is a zombie if and only if x is happy.

and these can be translated into the following English expressions

(6E) There exists something such that if it is a zombie, then it is happy.

(7E) Someone is either a zombie or happy.

(8E) Something is a zombie if and only if it is happy.

Notice that (6E) is a bit strange. We might exploit the fact that $Zx \rightarrow Hx$ is equivalent to $\neg Zx \vee Hx$ and translate (6+) instead:

(6) $(\exists x)(\neg Zx \vee Hx)$

(6E+) Something is either happy or not zombie.

Exercise 6: *Basic Translation in* **RL***; Key: $\mathcal{D}$: people; Px: x is poor; Lx: x is lazy; Rx: x is rich*

Involving the Universal Quantifier

1. $(\forall x)Px$
2. $(\forall x)(Px \rightarrow Lx)$
3. $(\forall x)Px \wedge (\forall x)Lx$, what is the difference between #3 and #2?
4. $(\forall x)(Px \wedge Lx)$
5. $(\forall x)(Px \rightarrow \neg Lx)$
6. $(\forall x)(Px \vee Lx)$

Involving the Existential Quantifier:

1. $(\exists x)Px$
2. $(\exists x)Px \land (\exists x)Rx$
3. $(\exists x)(Px \land Rx)$, what is the difference between #3 and #2?
4. $(\exists x)(Px \lor Rx)$
5. $\neg(\exists x)(Px \land Rx)$
6. $(\exists x)\neg(Px \land Rx)$

English to **RL**:

1. All poor people are lazy.
2. All lazy people are poor.
3. Not all lazy people are poor.
4. Some lazy person is not poor.
5. Someone is lazy and someone is poor.
6. If not all lazy people are poor, then not all poor people are lazy.

6.5.1 Translating Wffs with Overlapping Quantifiers

When dealing with wffs with quantifiers whose scope overlaps, does the order of the quantifiers matter? Consider the following eight wffs (let Lxy express the two-place English expression "x loves y")

1. $(\forall x)(\forall y)Lxy$
2. $(\forall y)(\forall x)Lxy$
3. $(\exists x)(\exists y)Lxy$
4. $(\exists y)(\exists x)Lxy$
5. $(\forall x)(\exists y)Lxy$
6. $(\exists y)(\forall x)Lxy$
7. $(\forall y)(\exists y)Lxy$
8. $(\exists y)(\forall y)Lxy$

While some of these wffs entail others, only the first two pairs of wffs are equivalent. That is, $(\forall x)(\forall y)Lxy$ is equivalent to $(\forall y)(\forall x)Lxy$ and $(\exists x)(\exists y)Lxy$ is equivalent to $(\exists y)(\exists x)Lxy$.

(1) and (2) express the proposition that "everyone loves everyone". In this scenario, every item in the domain of discourse loves every item in the domain of discourse. (3) and (4) express the proposition that "someone loves someone". In this scenario, at least one item in the discourse loves at least one other. In both cases, the order of the quantifiers does not impact the truth or falsity of the wff.

In contrast, (5)-(8) express different propositions. Let's characterize each in terms of a scenario. (5) is what I will call the "crush" scenario. It says that everyone loves at least one person. It does not say that everyone is loved (there may be some unloved individuals). What it says instead is that for any individual in the domain of discourse, that individual will love at least one other person. In other words, everyone has a crush on someone, even though not everyone is someone's crush.

(6) is what I will call the "Santa Claus scenario" (I need a better name). It says that there is at least one object who is loved by everyone. This expression is similar to (5) in that it implies that everyone loves at least one person. That is, in (5), every single person loves at least one person, but the loved person can differ from person to person. For example, in a scenario consisting of Jane, John, and Sally, (5) would be true if Jane loves John and John loves Jane and Sally loves herself. In contrast, (6) is true just in the case that there is one person loved by everyone, e.g. John loves Jane, Sally loves Jane, and Jane loves Jane.

(7) is what I will call the "Stalker scenario". It says that everyone is loved by someone. What this says is that if you go through the domain of discourse, pulling people one at a time, you will be able to find at least one other person who loves the selected person. So, if we consider Jane, John, and Sally, (7) is true in the case that, beginning with Jane, we can find at least one other person who loves Jane (e.g. John, but it could be anyone) and one person who loves John and one person who loves Sally. The person doing the loving need not be the same person in each case, nor is it the case that everyone loves someone. (7) differs from (5) in that it doesn't imply that everyone loves at least one other person. (7), in contrast, can be true if John loves Sally and Jane and himself, but neither Sally nor Jane love anyone. (7) also differs from (6) in that it doesn't imply that everyone loves at least one object.

(8) is what I will call the "Loving God scenario". It says that someone loves everyone. (8) is true provided there is at least one person who loves every single person in the domain. In contrast to (5), (8) does not imply that everyone loves at least one other person. Rather, it says that there is at least one person who loves all people. In contrast to (6), (8) does not imply that there is at least one person loved by all. It only says that there is one person who loves all. Finally, while (8) implies (7)–for if someone loves everyone, then everyone is loved by at least one person–(7) does not imply (8). This is because (7) can be true in a case where (8) is not, namely in the case where everyone is loved by someone, but everyone is not loved by a single person.

1.	$(\forall x)(\forall y)Lxy$	Everyone loves everyone.
2.	$(\forall y)(\forall x)Lxy$	Everyone loves everyone.
3.	$(\exists x)(\exists y)Lxy$	Someone loves someone.
4.	$(\exists y)(\exists x)Lxy$	Someone loves someone.
5.	$(\forall x)(\exists y)Lxy$	Everyone loves someone.
6.	$(\exists y)(\forall x)Lxy$	Someone is loved by everyone.
7.	$(\forall y)(\exists y)Lxy$	Everyone is loved by someone.
8.	$(\exists y)(\forall y)Lxy$	Someone loves everyone.

Questions

1. Know all of the *symbols* of **RL**, viz., which symbols are names, variables, truth-functional operators, quantifiers
2. Know how to determine if a formula is a well-formed formula. For

example, which of the following are wffs (where Px is a one-place predicate and Rxy is a two-place predicate): Pa, Pab, $(\forall x)Px$, $(\exists x)Rxx$, $\neg\neg Pa$, $(\forall x)(\forall y)Rxy$, Px, Rxy

3. Know the definition of a free variable, a bound variable, an open formula, a closed formula.
4. For any well-formed formula, know how to identify whether it has any free or bound variables and whether the formula is open or closed.
5. Given a model, know how to determine whether a closed wff is true or false. For example, if $\mathcal{D} = \{a, b, c, d, e\}, \mathscr{I}(a) = a, \mathscr{I}(b) = b, \mathscr{I}(c) = c, \mathscr{I}(d) = d, \mathscr{I}(e) = e, \mathscr{I}(P) = \{a, b, c, d\}, \mathscr{I}(R) = \{e\}$, determine whether the following wffs are true or false:
   - Pa
   - $(\forall x)Px$
   - $Pa \wedge Pb$
   - $(\exists x)Px$
   - $(\exists x)\neg Px$
6. Given a translation key, be able to translate an English sentence into a wff in **RL**.
7. In comparison to **RL**, what is the principal weakness of **PL** as a formal system?
8. What is a model in **RL**?
9. What is an interpretation in **RL**?