

Definition 1: Conjunction Introduction ($\wedge I$)

$\phi, \psi \vdash \phi \wedge \psi$ or $\phi, \psi \vdash \psi \wedge \phi$

Definition 2: Conjunction Elimination ($\wedge E$)

$\phi \wedge \psi \vdash \phi$ or $\phi \wedge \psi \vdash \psi$

Definition 3: Conditional Elimination ($\rightarrow E$)

$\phi \rightarrow \psi, \phi \vdash \psi$

Definition 4: Conditional Introduction ($\rightarrow I$)

n	ϕ	A
	$\vdots$	
m	ψ	
m+1	$\phi \rightarrow \psi$	$\rightarrow I$ n-m

Definition 5: Negation Introduction ($\neg I$)

k	ϕ	A
	$\vdots$	
n	ψ	
n+1	$\neg(\psi)$	
n+2	$\neg(\phi)$	$\neg I$ k-(n+1)

Definition 6: Negation Elimination ($\neg E$)

k	$\neg(\phi)$	A
	$\vdots$	
n	ψ	
n+1	$\neg(\psi)$	
n+2	ϕ	$\neg E$ k-(n+1)

Definition 7: Disjunction Introduction ($\vee I$)

$\phi \vdash \phi \vee \psi$ or $\phi \vdash \psi \vee \phi$

Definition 8: Disjunction Elimination ($\vee E$)

n	$\phi \vee \psi$	P
k	ϕ	A
	$\vdots$	
k+i	χ	
j	ψ	A
	$\vdots$	
j+1	χ	
m	χ	$\vee E$ n, k-(k+i), j-(j+1)

Definition 9: Biconditional Introduction ($\leftrightarrow I$)

n	ϕ	A
	$\vdots$	
n+i	ψ	
k	ψ	A
	$\vdots$	
k+j	ϕ	
m	$\phi \leftrightarrow \psi$	$\leftrightarrow I$ n+1, k+j

Definition 10: Biconditional Elimination ($\leftrightarrow E$)

$\phi \leftrightarrow \psi, \phi \vdash \psi$ or $\phi \leftrightarrow \psi, \psi \vdash \phi$

Definition 11: Reiteration (R)

$\phi \vdash \phi$

Definition 12: Disjunctive Syllogism (DS)

$\phi \vee \psi, \neg(\psi) \vdash \phi$ or $\phi \vee \psi, \neg(\phi) \vdash \psi$

Definition 13: Modus Tollens (MT)

$\phi \rightarrow \psi, \neg(\psi) \vdash \neg(\phi)$

Definition 14: Hypothetical Syllogism (HS)

$\phi \rightarrow \psi, \psi \rightarrow \chi \vdash \phi \rightarrow \chi$

Definition 15: Double Negation (DN)

$\phi \dashv\vdash \neg\neg(\phi)$

Definition 16: De Morgan's Laws (DeM)

$\neg(\phi \vee \psi) \dashv\vdash \neg(\phi) \wedge \neg(\psi)$ or $\neg(\phi \wedge \psi) \dashv\vdash \neg(\phi) \vee \neg(\psi)$

Definition 17: Implication (IMP)

$\phi \rightarrow \psi \dashv\vdash \neg(\phi) \vee \psi$