

Directions: The following exam consists of 10 questions, for a total of 100 points. Read each question carefully (note: answers may break onto the next page). This exam tests your knowledge over the material from Chapter 5 of the course text, lectures, videos, handouts, and discussion. You may write on the test itself, but place final answers on the “answer sheet” (last page) provided.

1 Proofs

Directions: Provide proofs for the following syntactic entailments. Be sure to setup the proof correctly, number all lines, and clearly indicate how each line is justified using the rules from the deductive apparatus.

1. (15 points) $Q \vee Z, M \wedge (Q \wedge R), \neg B \wedge C \vdash (R \wedge \neg B) \wedge M$
2. (15 points) $(A \wedge \neg Z) \rightarrow Q, S \wedge \neg Z, A \vdash Q \vee \neg \neg W$
3. (15 points) $C \vee S, C \rightarrow (G \wedge F), S \rightarrow (G \wedge \neg L) \vdash G$
4. (15 points) $\neg(Z \rightarrow W), S \rightarrow W \vdash \neg S$
5. (15 points) $\vdash (R \rightarrow M) \vee \neg(P \wedge \neg P)$
6. (15 points) $\vdash (P \wedge \neg S) \rightarrow ((\neg Q \vee S) \rightarrow \neg Q)$

Directions: Translate the following arguments and then create a proof for it.

7. (10 points) If God is real and God is good, then there is no evil in the world. There is evil in the world. Therefore, God is not real or God is not good. **This is the end of the exam.**

2 Extra-Credit

8. (3 points (bonus)) Absorption law for disjunction: $P \vee (P \wedge Q) \dashv\vdash P$. Solve only using intelim rules.
9. (3 points (bonus)) Commutation for disjunction: $P \vee Q \dashv\vdash Q \vee P$. Solve only using intelim rules.
10. (10 points (bonus)) Let's define two rules. First, $\oplus E1$ is defined as follows: $\phi \oplus \psi, \phi \vdash \neg \psi$ or $\phi \oplus \psi, \psi \vdash \neg \phi$. Next, $\oplus E2$ is defined as follows: $\phi \oplus \psi, \neg(\psi) \vdash \phi$ or $\phi \oplus \psi, \neg(\phi) \vdash \psi$. Now prove the following:
 $A \oplus B \vdash (A \vee B) \wedge \neg(A \wedge B)$



Derivation rules

Definition 2.0: Conjunction Introduction ($\wedge I$)

$\phi, \psi \vdash \phi \wedge \psi$ or $\phi, \psi \vdash \psi \wedge \phi$

Definition 2.0: Conjunction Elimination ($\wedge E$)

$\phi \wedge \psi \vdash \phi$ or $\phi \wedge \psi \vdash \psi$

Definition 2.0: Conditional Introduction ($\rightarrow I$)

n	ϕ	A
	\vdots	
m	ψ	
$m+1$	$\phi \rightarrow \psi$	$\rightarrow I \ n-m$

Definition 2.0: Conditional Elimination ($\rightarrow E$)

$\phi \rightarrow \psi, \phi \vdash \psi$

Definition 2.0: Reiteration (R)

$\phi \vdash \phi$

Definition 2.0: Negation Introduction ($\neg I$)

k	ϕ	A
	\vdots	
n	ψ	
$n+1$	$\neg(\psi)$	
$n+2$	$\neg(\phi)$	$\neg I \ k-(n+1)$

Definition 2.0: Negation Elimination ($\neg E$)

k	$\neg(\phi)$	A
	\vdots	
n	ψ	
$n+1$	$\neg(\psi)$	
$n+2$	ϕ	$\neg E \ k-(n+1)$

Definition 2.0: Disjunction Introduction ($\vee I$)

$\phi \vdash \phi \vee \psi$ or $\phi \vdash \psi \vee \phi$

Definition 2.0: Disjunction Elimination ($\vee E$)

n	$\phi \vee \psi$	P
k	ψ	A
	\vdots	
$k+i$	χ	
j	ψ	A
	\vdots	
$j+1$	χ	
m	χ	$\vee E \ n, k-(k+i), j-(j+1)$

Definition 2.0: Biconditional Introduction ($\leftrightarrow I$)

n	ϕ	A
	\vdots	
$n+i$	ψ	
k	ψ	A
	\vdots	
$k+j$	ϕ	
m	$\phi \leftrightarrow \psi$	$\leftrightarrow I \ n+1, k+j$

Definition 2.0: Biconditional Elimination ($\leftrightarrow E$)

$\phi \leftrightarrow \psi, \phi \vdash \psi$ or $\phi \leftrightarrow \psi, \psi \vdash \phi$

Definition 2.0: Disjunctive Syllogism (DS)

$\phi \vee \psi, \neg(\psi) \vdash \phi$ or $\phi \vee \psi, \neg(\phi) \vdash \psi$

Definition 2.0: Modus Tollens (MT)

$\phi \rightarrow \psi, \neg(\psi) \vdash \neg(\phi)$

Definition 2.0: Hypothetical Syllogism (HS)

$\phi \rightarrow \psi, \psi \rightarrow \chi \vdash \phi \rightarrow \chi$

Definition 2.0: Double Negation (DN)

$\phi \dashv\vdash \neg\neg(\phi)$



Definition 2.0: De Morgan's Laws (DeM)

$\neg(\phi \vee \psi) \dashv\vdash \neg(\phi) \wedge \neg(\psi)$ or $\neg(\phi \wedge \psi) \dashv\vdash \neg(\phi) \vee \neg(\psi)$

Definition 2.0: Implication (IMP)

$\phi \rightarrow \psi \dashv\vdash \neg(\phi) \vee \psi$



Solutions for plproofs/su26a

1. — Answer: $Q \vee Z, M \wedge (Q \wedge R), \neg B \wedge C \vdash (R \wedge \neg B) \wedge M$

1	$Q \vee Z$	P
2	$M \wedge (Q \wedge R)$	P
3	$\neg B \wedge C$	P, $(R \wedge \neg B) \wedge M$
4	$\neg B$	$\wedge E, 3$
5	$Q \wedge R$	$\wedge E, 2$
6	R	$\wedge E, 5$
7	M	$\wedge E, 2$
8	$R \wedge \neg B$	$\wedge I, 6, 4$
9	$(R \wedge \neg B) \wedge M$	$\wedge I, 7, 8$

2. — Answer: $(A \wedge \neg Z) \rightarrow Q, S \wedge \neg Z, A \vdash Q \vee \neg \neg W$

1	$(A \wedge \neg Z) \rightarrow Q$	P
2	$S \wedge \neg Z$	P
3	A	P, $Q \vee W$
4	$\neg Z$	$\wedge E, 2$
5	$A \wedge \neg Z$	$\wedge I, 3, 4$
6	Q	$\rightarrow E, 1, 5$
7	$Q \vee \neg \neg W$	$\vee I, 6$

3. — Answer: $C \vee S, C \rightarrow (G \wedge F), S \rightarrow (G \wedge \neg L) \vdash G$

1	$C \vee S$	P
2	$C \rightarrow (G \wedge F)$	P
3	$S \rightarrow (G \wedge \neg L)$	P, G
4	C	A
5	$G \wedge F$	$\rightarrow E, 2, 4$
6	G	$\wedge E, 5$
7	S	A
8	$G \wedge \neg L$	$\rightarrow E, 3, 7$
9	G	$\wedge E, 8$
10	G	$\vee E, 1, 4-6, 7-9$

4. — Answer: $\neg(Z \rightarrow W), S \rightarrow W \vdash \neg S$

1	$\neg(Z \rightarrow W)$	P
2	$S \rightarrow W$	P, $\neg S$
3	$\neg(\neg Z \vee W)$	1 <i>IMP</i>
4	$\neg \neg Z \wedge \neg W$	3 <i>DEM</i>
5	$\neg W$	4 $\wedge E$
6	$\neg S$	2, 5 <i>MT</i>

5. — Answer: $\vdash (R \rightarrow M) \vee \neg(P \wedge \neg P)$

1	$(P \wedge \neg P)$	A
2	P	1 $\wedge E$
3	$\neg P$	1 $\wedge E$



- 4 $\neg(P \wedge \neg P)$ 1-3 $\neg I$
 5 $(R \rightarrow M) \vee \neg(P \wedge \neg P)$ 4 $\vee I$
 6. — Answer: $\vdash (P \wedge \neg S) \rightarrow ((\neg Q \vee S) \rightarrow \neg Q)$

1	$P \wedge \neg S$	A
2	$\neg Q \vee S$	A
3	$\neg S$	1 $\wedge E$
4	$\neg Q$	2,3 DS
5	$(\neg Q \vee S) \rightarrow \neg Q$	2-4 $\rightarrow I$
6	$(P \wedge \neg S) \rightarrow ((\neg Q \vee S) \rightarrow \neg Q)$	1-5 $\rightarrow I$

7. Translation: $(R \wedge G) \rightarrow \neg E, E \vdash \neg R \vee \neg G$.

- 1 $(R \wedge G) \rightarrow \neg E$ P
 2 E P, $\neg R \vee \neg G$
 3 $\neg(R \wedge G)$ 1,2 MT
 4 $\neg R \vee \neg G$ 3 DEM

8. Tips. Proof 1: $P \vee (P \wedge Q) \vdash P$. Use $\vee E$. Proof 2: $P \vee (P \wedge Q) \dashv P$. Just use $\vee I$
 9. Tips. Just use $\vee E$ for both proofs
 10.

1	$P \oplus Q$	P, $(P \vee Q) \wedge \neg(P \wedge Q)$
2	$\neg((P \vee Q) \wedge \neg(P \wedge Q))$	A
3	$\neg(P \vee Q) \vee \neg\neg(P \wedge Q)$	2 DeM
4	$\neg P$	A
5	$\neg P \vee \neg Q$	3 $\vee I$
6	$\neg(P \wedge Q)$	4 DeM
7	$\neg(P \vee Q)$	3,6 DS
8	$\neg P \wedge \neg Q$	7 DeM
9	$\neg P$	8 $\wedge E$
10	$\neg Q$	8 $\wedge E$
11	Q	1,10 $\oplus E2$
12	P	4-11 $\neg E$
13	$\neg Q$	1,12 $\oplus E1$
14	$P \vee Q$	12 $\vee I$
15	$\neg\neg(P \wedge Q)$	3,14 DS
16	$P \wedge Q$	15 DN
17	Q	$\wedge E$
18	$(P \vee Q) \wedge \neg(P \wedge Q)$	2-17 $\neg I$

