

**Directions:** The following exam consists of 24 questions, for a total of 100 points. Read each question carefully (note: answers may break onto the next page). This exam tests your knowledge over the material from Chapter 3 and Chapter 4 of the course text and lectures. You may write on the test itself, but place final answers on the “answer sheet” (last page) provided.

## 1 Definitions, Concepts, and Basic Mechanics

1. (2 points) What advantage does the truth table and truth tree tests have over the imagination test for validity?
  - A. the truth table/tree tests are poetic; they take into account the spirit of human nature
  - B. If an argument is persuasive in English, then the table/tree methods will tell us whether we ought to be persuaded by them.
  - C. If an argument is deductively valid in English, then the truth table/tree method will always correctly determine whether it is (in fact) valid in the language of propositional logic (PL).
  - D. The truth table/tree tests are mechanical (decision procedures)
2. (2 points) What is one advantage of truth trees over truth tables?
  - A. trees provide the user a more graphical way of seeing the truth or falsity of an argument, specifically by showing whether an argument is true or false under every interpretation
  - B. Tree trees use a node-branch (or tree) structure to test arguments for validity while tables use a table.
  - C. If an argument is translated into PL, a table can show that a set of wffs is consistent but a tree cannot.
  - D. In contrast to truth tables where the complexity of the table is a function of the number of propositional letters (more letters, more rows required), the complexity of a truth tree is not a function of the number of propositional letters.
3. (2 points) A set of wffs  $\Gamma$  semantically entails a wff  $\phi$  if and only if what? That is,  $\Gamma \models \phi$  if and only if what?
  - A. there is **an** interpretation such that each of the members (wffs) of  $\Gamma$  are true and  $\phi$  is false.
  - B. there is **at least two** interpretations such that each of the members (wffs) of  $\Gamma$  are true and  $\phi$  is false.
  - C. there is **no** interpretation such that each of the members (wffs) of  $\Gamma$  are false and  $\phi$  is true.
  - D. there is **no** interpretation such that each of the members (wffs) of  $\Gamma$  are true and  $\phi$  is false.

## 2 Determining the truth of wffs

**Directions:** Determine the truth value (write T or F on the answersheet). *Note that in some cases you don't need to know all of the truth values for some (or all) of the propositional letters.*

4. (2 points) Suppose  $\mathcal{I}(R) = F$ , what is  $v(P \wedge \neg Q) \vee \neg R$ ?
5. (2 points) What is the truth value of  $A \vee \neg A$ ?
6. (2 points) What is the truth value of  $\neg Z \rightarrow \neg Z$ ?
7. (2 points) Suppose  $\mathcal{I}(Q) = F$ , what is  $v(P \rightarrow (Q \rightarrow P))$ ?

## 3 Truth-tree decomposition rules



**Directions:** Write the abbreviation (e.g.  $\wedge D$ ) for the decomposition rule that should be used on wffs below. Indicate only the *first* decomposition rule that would be used.

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| 8. (2 points) $\neg B \wedge \neg Q?$      | 15. (2 points) $\neg(A \rightarrow \neg R)$              |
| 9. (2 points) $\neg\neg A \vee \neg Z$     | 16. (2 points) $\neg\neg(B \wedge Q)$                    |
| 10. (2 points) $(Q \vee \neg L) \wedge M$  | 17. (2 points) $\neg(A \wedge R)$                        |
| 11. (2 points) $\neg\neg S$                | 18. (2 points) $\neg(B \leftrightarrow \neg\neg C)$      |
| 12. (2 points) $A \rightarrow \neg R$      | 19. (2 points) $\neg\neg(Z \leftrightarrow \neg\neg S)?$ |
| 13. (2 points) $\neg C \rightarrow \neg Z$ | 20. (2 points) $\neg\neg Z \leftrightarrow \neg M?$      |
| 14. (2 points) $\neg A \leftrightarrow X$  |  |

## 4 Truth table construction

**Directions:** On the answer sheet, construct a **truth table** that tests for the indicated property. To receive full credit, you must (1) construct the entire truth table (each row and each T and F), (2) label whether the table indicates the property in question (e.g., "tautology"), and (3) clearly explain why the table indicates the property in question (e.g., "The table shows  $\phi$  is a tautology because ..."). Be sure (1) the table is fully complete (do not skip steps) and (2) Ts and Fs are clearly distinguishable (you can use 1 or 0 if it is easier).

21. (15 points) Determine whether  $A \rightarrow B, \neg B \vdash \neg A$  is a valid sequent.
22. (15 points) Determine whether  $B \rightarrow \neg(Z \vee \neg B)$  is a contingency, tautology, or contradiction.

## 5 Truth tree construction

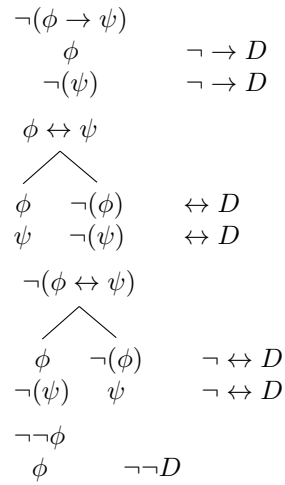
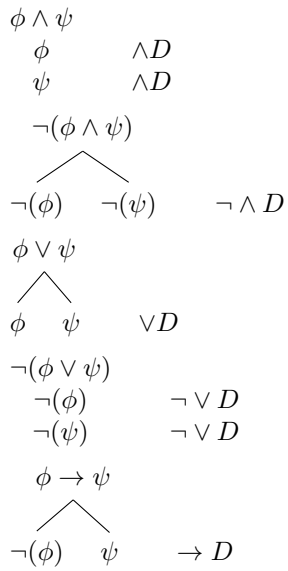
**Directions:** On the answer sheet, construct a **truth tree** that tests for the indicated property. To receive full credit, you must (1) construct the entire tree (numbering, the tree, and the node justification), (2) label whether the tree indicates the property in question (e.g., "tautology"), and (3) if an interpretation can be recovered from the tree, provide that interpretation.

23. (15 points) Determine whether  $(B \rightarrow B) \wedge \neg(D \vee \neg Q)$  is a contingency, tautology, or contradiction.
24. (15 points) On the answer sheet, construct either a **truth table** or a **truth tree** for the following argument:  $A \wedge (B \wedge C), A \rightarrow B, \neg B \vee C \models C \wedge \neg D$ . To receive full credit, you must (i) construct the *entire* truth table or tree, (ii) label whether it is a valid or invalid (that is, entailment of non-entailment), and (iii) if the argument is invalid, identify write out the interpretation (e.g.  $\mathcal{I}(P) = T, \mathcal{I}(Q) = F$ ) demonstrating its invalidity.



$P$	$\neg P$
$T$	$F$
$F$	$T$

$P$	$R$	$P \wedge R$	$P \vee R$	$P \rightarrow R$	$P \leftrightarrow R$
$T$	$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$F$	$F$
$F$	$T$	$F$	$T$	$T$	$F$
$F$	$F$	$F$	$F$	$T$	$T$



## Solutions for pltablestrees/su26a

1. D
2. D
3. D
4. T
5. T
6. T
7. T
8.  $\wedge D$
9.  $\vee D$
10.  $\wedge D$
11.  $\neg\neg D$
12.  $\rightarrow D$
13.  $\rightarrow D$
14.  $\leftrightarrow D$
15.  $\neg\rightarrow D$
16.  $\neg\neg D$
17.  $\neg\wedge D$
18.  $\neg\leftrightarrow D$
19.  $\neg\neg D$
20.  $\leftrightarrow D$

21.  $A \rightarrow B, \neg B \vdash \neg A$  is a valid sequent

A	B	$A \rightarrow B$	$\neg B$	$(A \rightarrow B) \wedge \neg B$	$\neg A$
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

22. The wff is a contingency. Notice that under the main operator (the right arrow), there is at least one T and at least one F

B	Q	B	$\rightarrow$	$\neg$	(Z	$\vee$	$\neg$	B)
T	T	T	F	F	T	T	F	T
T	F	T	T	T	F	F	F	T
F	T	F	T	F	T	T	T	F
F	F	F	T	F	F	T	T	F

23. Answer may vary, but the tree for this wff shows the wff is a contingency. It is a contingency since the tree test for contradiction shows that it is not a contradiction (all branches close), while the tree test for tautology shows that it is not a tautology (since the negated version of the wff yields a tree where all branches close). Since a wff is exactly one of the following (contingency, tautology, contradiction), the wff is a contingency.

Test for contradiction:

1.  $(B \rightarrow B) \wedge \neg(D \vee \neg Q) \checkmark$  P
2.  $(B \rightarrow B) \checkmark$   $1 \wedge D$
3.  $\neg(D \vee \neg Q)$   $1 \wedge D$
4.  $\neg D$   $3 \neg \wedge D$
5.  $\neg Q$   $3 \neg \wedge D$
6.  $\neg B$   $B$   $2 \rightarrow D$

Test for tautology:



1.	$\neg((B \rightarrow B) \wedge \neg(D \vee \neg Q)) \checkmark$	P
	$\swarrow \quad \searrow$	
2.	$\neg(B \rightarrow B) \checkmark$	$\neg\neg(D \vee \neg Q) \checkmark$
3.	$B$	$D \vee \neg Q \checkmark$
4.	$\neg B$	$\swarrow \quad \searrow$
5.		$D \quad \neg Q$

24. The argument is invalid or a case of non-entailment. If creating a table, then a table of at least 16 rows is required and the table must provide a completed row where the premises are true and the conclusion is false. In the case of a tree, the following is a possible answer:

1.	$A \wedge (B \wedge C) \checkmark$	P
2.	$A \rightarrow B \checkmark$	P
3.	$\neg B \vee C \checkmark$	P
4.	$\neg(C \wedge \neg D) \checkmark$	P
5.	$A$	$1 \wedge D$
6.	$B \wedge C \checkmark$	$1 \wedge D$
7.	$B$	$5 \wedge D$
8.	$C$	$5 \wedge D$
	$\swarrow \quad \searrow$	
9.	$\neg B \quad C$	$3 \vee D$
	$\otimes$	
	6,8	
10.	$\neg C \quad \neg\neg D$	$4 \neg \wedge D$
	$\swarrow \quad \searrow$	
	$\otimes$	
	9,8	
11.	$\neg A \quad B$	$2 \rightarrow D$
12.	$\otimes \quad D$	$10 \neg\neg D$
	4,11	

Since it is invalid, an interpretation must be provided. Here is an interpretation:  $\mathcal{I}(A) = T, \mathcal{I}(B) = T, \mathcal{I}(C) = F, \mathcal{I}(D) = T$

