Directions: This exam has 34 questions, for a total of 100 points and 0 bonus points. Place proofs on the blank space on the answersheet. Good luck!

1. (2 points) A model M is a two-part structure consisting of what? Select two choices.
   A. A valuation function v
   B. A set of second-order predicates that can be quantified over, e.g., $(\forall P)Pa$
   C. A domain $\mathcal{D}$
   D. An interpretation function $\mathscr{I}$
2. (2 points) An interpretation of QL is a function that does what (indicate all that apply):
   A. specifies what objects are in the domain.
   B. assigns truth values to n-place predicate terms followed by n terms.
   C. for each name in QL it assigns that name one and only one item in $\mathcal{D}$
   D. for each n-place predicate term in QL assigns, it assigns that predicate term a set of n-tuples composed of elements from $\mathcal{D}$
3. (2 points) What is a derivation / proof of ψ from Γ using **QD**?
   A. a finite string of wffs starting with some premises Γ and ending with ψ.
   B. a finite string of wffs starting with some premises Γ or assumptions and ending with ψ.
   C. a finite string of formulas from a set Γ of QL wffs where (i) the last formula in the string is ψ and (ii) each formula is either a premise, an assumption, or is the result of the preceding formulas and the deductive apparatus.

Directions: Are the following wffs open or closed?

KEY: (A) = Open. (B) = Closed.

4. (2 points) $\neg Fab$
5. (2 points) $\neg Qy$
6. (2 points) $(\exists x)(\forall y)\neg Lxy$

Directions: Determine whether the following wffs are T or F by using the following model: $\mathcal{D} = \{1, 2, 3\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(N) = \{1, 2, 3\}$, $\mathscr{I}(G) = \{\langle 2, 1\rangle, \langle 3, 2\rangle, \langle 3, 1\rangle\}$, $\mathscr{I}(I) = \varnothing$, $\mathscr{I}(E) = \{2\}$, $\mathscr{I}(O) = \{1, 3\}$

KEY: (A) = T. (B) = F.

7. (2 points) $(\exists x)\neg Ix$
8. (2 points) Gab
9. (2 points) Gba
10. (2 points) $(\exists y)Ey$
11. (2 points) $(\exists y)\neg Ey$
12. (2 points) $\neg(\exists y)Ey$
13. (2 points) $(\forall x)Ox$
14. (2 points) $(\exists x)(Nx \wedge Gxc)$
15. (2 points) $(\exists x)(Ex \wedge Nx)$
16. (2 points) $(\forall y)(Ey \rightarrow Nx)$

Directions: Select the correct translation of the QL wffs below using the following: Ax: x is an athlete. Sx: x is a student.

KEY: (A) = Some athletes are students. (B) = Some athletes are not students. (C) = All students are athletes. (D) = All athletes are students. (E) = No athletes are students.

17. (2 points) $(\exists x)(Ax \wedge Sx)$
18. (2 points) $(\forall x)(Ax \rightarrow \neg Sx)$
19. (2 points) $(\exists x)(Ax \wedge \neg Sx)$
20. (2 points) $(\forall x)(Ax \rightarrow Sx)$
21. (2 points) $(\forall x)(Sx \rightarrow Ax)$
22. (2 points) $\neg(\exists x)(Ax \wedge Sx)$

Directions: Select the correct translation of the QL wffs below using the following: a: Ava; Lxy: x loves y.

KEY: (A) = Someone loves Ava. (B) = Ava loves someone. (C) = Someone loves someone. (D) = Someone loves themselves.

23. (2 points) $(\exists x)Lxa$
24. (2 points) $(\exists x)Lax$
25. (2 points) $(\exists x)(\exists y)Lxy$
26. (2 points) $(\exists x)Lxx$

Directions: Select the derivation rule described in the following:

KEY: (A) = $\exists I$, (B) = $\exists E$, (C) = $\forall I$, (D) = $\forall I$, (E) = QN

27. (2 points) $\neg Qab \wedge Pa \vdash (\exists z)(\neg Qzb \wedge Pz)$
28. (2 points) $(\forall z)(Fz \wedge \neg Fz) \vdash Fd \rightarrow \neg Fd$
29. (2 points) From $Qc \wedge Fc$ to $(\forall y)(Qy \wedge Fy)$ provided (1) c is not in a premise or in an assumption of an active subproof and (2) c is not in $(\forall y)(Qy \wedge Fy)$?
30. (2 points) $\neg(\forall z)Fz \vdash (\exists z)\neg Fz$

Directions: Provide proofs for the following:

31. (10 points) $(\exists x)Fx \rightarrow (\forall y)By, Fa \vdash Ba$
32. (10 points) $Faa, (\forall x)(\forall y)Lxy \vdash (\exists z)(\exists x)Lzx$
33. (10 points) $(\exists x)\neg Mx \vdash (\exists y)(My \vee \neg Py)$
34. (10 points) $\vdash (\forall x)(Lxx \rightarrow (Gx \rightarrow Lxx))$

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Solutions for qlproofs/exam4sample.tex

1. C, (D)
2. C, (D)
3. C
4. closed
5. open
6. closed
7. T
8. F
9. T
10. T
11. T
12. F
13. F
14. F
15. T
16. T
17. A
18. E
19. B
20. D
21. C
22. E
23. Someone loves Ava.
24. Ava loves someone.
25. Someone loves someone.
26. Someone loves themselves.
27. $\exists I$
28. $\forall E$
29. $\forall I$
30. QN
31. $(\exists x)Fx \to (\forall y)By, Fa \vdash Ba$

1	$(\exists x)Fx \to (\forall y)By$	P
2	Fa	P
3	$(\exists x)Fx$	2$\exists I$
4	$(\forall y)By$	1,3$\to E$
5	Ba	4$\forall E$

32. $Faa, (\forall x)(\forall y)Lxy \vdash (\exists z)(\exists x)Lzx$

1	Faa	P
2	$(\forall x)(\forall y)Lxy$	P
3	$(\forall y)Lay$	2$\forall E$
4	Laa	3$\forall E$
5	$(\exists x)Lax$	4$\exists I$
6	$(\exists z)(\exists x)Lzx$	5$\exists I$

33. $(\exists x)\neg Mx \vdash (\exists y)(My \vee \neg Py)$

1	$(\exists x)\neg Mx$	P
2	$\quad\vert\underline{\neg Ma}$	A

3	$\neg Ma \vee \neg Pa$	$2\vee I$
4	$(\exists y)(My \vee \neg Py)$	$3\exists I$
5	$(\exists y)(My \vee \neg Py)$	1, 2-4 $\exists E$

34. $\vdash (\forall x)(Lxx \to (Gx \to Lxx))$

1	$\underline{Laa}$	A
2	$\quad \underline{Ga}$	A
3	$\quad Laa$	1 R
4	$Ga \to Laa$	$2\text{-}3\to I$
5	$Laa \to (Ga \to Laa)$	$1\text{-}4\to I$
6	$(\forall x)(Lxx \to (Gx \to Lxx))$	$5\forall I$