Directions: The following exam consists of 30 questions, for a total of 100 points. Read each question carefully (note: answers may break onto the next page). This exam tests your knowledge over the material from Chapter 5 of the course text, lectures, videos, handouts, and discussion. You may write on the test itself, but place final answers on the "answer sheet" (last page) provided.

1 Concepts and terminology

1. (2 points) What is a derivation of ϕ from Γ in the language of propositional logic (PL)?
   A. A derivation of ϕ is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with ϕ.
   B. A derivation of ϕ is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with ϕ.
   C. A derivation of ϕ is an *infinite* string of formulas from a set Γ of PL wffs where (i) the last formula in the string is ϕ and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.
   D. A derivation of ϕ is a *finite* string of formulas from a set Γ of PL wffs where (i) the last formula in the string is ϕ and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.

2. (2 points) What is a deductive apparatus for PL?
   A. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.
   B. It is a set of rules of reason that all people use to reason from one proposition to another, including, but not limited to, hypothetical and probabilistic reasoning.
   C. a set of rules that state that the rows in a proof need to be numbered.
   D. a set of derivation rules that express which wffs ϕ can be written after which wffs ψ in a derivation.

3. (2 points) In logic, if an argument is said to be "good", then "its conclusion follows from its premises". One way to make this idea precise is to say that "it is impossible for the premises to be true and the conclusion false". What is the other way?
   A. if you can prove that the conclusion follows from the premises
   B. if you can construct a truth table showing that the conclusion follows from the premises.

2 Proofs

Directions: What derivation rule that permits the step in the proof below.

KEY: (A) = $\wedge E$, (B) = $\wedge I$, (C) = $\rightarrow E$, (D) $\rightarrow I$, (E) = R

4. (2 points) Under the assumption of A, B follows. Therefore, if A, then B.

5. (2 points) $A \wedge \neg P \vdash \neg P$

6. (2 points) $\neg(A \vee \neg X) \wedge \neg Q \vdash \neg(A \vee \neg X)$

7. (2 points) $\neg X \vdash \neg X$

8. (2 points) $\neg Z \rightarrow S, \neg Z \vdash S$

KEY: (A) = $\leftrightarrow E$, (B) = $\leftrightarrow I$, (C) = $\vee E$, (D) $\vee I$, (E) = R

9. (2 points) A if and only if B. A. Therefore, B.
10. (2 points) A or B. Assuming A is the case, it follows that C is the case.. And, assuming B is the case, it follows that C is the case. Therefore, C is the case.
11. (2 points) What rule allows you to derive $\neg P$ from (1) an assumption P and (2) Z and $\neg Z$ within the subproof started by P?
12. (2 points) $(L \rightarrow Q) \leftrightarrow M,\ M \vdash L \rightarrow Q$
13. (2 points) $\neg F \wedge R \vdash (\neg F \wedge R) \vee L$
14. (2 points) From $\neg Z \vee C$ and two subproofs D is derived. The first subproof is where $\neg Z$ is assumed and D is derived. The second subproof is where C is assumed and D is derived.
15. (2 points) $W \vee M \vdash W \vee \neg\neg M$

KEY: (A) = MT, (B) = IMP, (C) = DeM, (D) DS, (E) = HS

16. (2 points) A or B. Not B. Therefore A.
17. (2 points) $Z \rightarrow Q, \neg Q \vdash \neg Z$
18. (2 points) $Z \rightarrow B \vdash \neg Z \vee B$
19. (2 points) If A, then B. Not-B. Therefore, not-A.
20. (2 points) $\neg(Z \vee B) \vdash \neg Z \wedge \neg B$
21. (2 points) $\neg M \vee P, \neg P \vdash \neg M$
22. (2 points) Not both A and B. Therefore, not-A or not-B.
23. (2 points) $Q \rightarrow \neg Z, \neg Z \rightarrow M \vdash Q \rightarrow M$
24. (2 points) $(S \wedge T) \rightarrow L, \neg L \vdash \neg(S \wedge T)$
25. (2 points) If A then B. If B then C. Therefore, if A then C.

Directions: Provide proofs for the following syntactic entailments. Be sure to setup the proof correctly, number all lines, and clearly indicate how each line is justified using the rules from the deductive apparatus.

26. (10 points) $Q \vee Z, M \wedge (Q \wedge R), \neg B \wedge C \vdash (R \wedge \neg B) \wedge M$
27. (10 points) $(A \wedge \neg Z) \rightarrow Q, S \wedge \neg Z, A \vdash Q \vee \neg\neg W$
28. (10 points) $C \vee S, C \rightarrow (G \wedge F), S \rightarrow (G \wedge \neg L) \vdash G$
29. (10 points) $\vdash (R \rightarrow M) \vee \neg(P \wedge \neg P)$
30. (10 points) $\vdash (P \wedge \neg S) \rightarrow ((\neg Q \vee S) \rightarrow \neg Q)$

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Solutions for plproofs/exam3sample.tex

1. D
2. D
3. A
4. $\to I$
5. $\wedge E$
6. $\wedge E$
7. R
8. $\to E$
9. $\leftrightarrow E$
10. $\vee E$, ($\leftrightarrow E$)
11. $\neg I$
12. $\leftrightarrow E$
13. $\vee I$
14. $\vee E$
15. DN
16. DS
17. MT
18. IMP
19. MT
20. DeM
21. DS
22. DeM
23. HS
24. MT
25. HS
26. — Answer: $Q \vee Z, M \wedge (Q \wedge R), \neg B \wedge C \vdash (R \wedge \neg B) \wedge M$

1	$Q \vee Z$	P
2	$M \wedge (Q \wedge R)$	P
3	$\neg B \wedge C$	P, $(R \wedge \neg B) \wedge M$
4	$\neg B$	$\wedge E$, 3
5	$Q \wedge R$	$\wedge E$, 2
6	R	$\wedge E$, 5
7	M	$\wedge E$, 2
8	$R \wedge \neg B$	$\wedge I$, 6,4
9	$(R \wedge \neg B) \wedge M$	$\wedge I$, 7,8

27. — Answer: $(A \wedge \neg Z) \to Q, S \wedge \neg Z, A \vdash Q \vee \neg\neg W$

1	$(A \wedge \neg Z) \to Q$	P
2	$S \wedge \neg Z$	P
3	A	P, $Q \vee W$
4	$\neg Z$	$\wedge E$, 2
5	$A \wedge \neg Z$	$\wedge I$, 3, 4
6	Q	$\to E$, 1, 5
7	$Q \vee \neg\neg W$	$\vee I$, 6

28. — Answer: $C \vee S, C \to (G \wedge F), S \to (G \wedge \neg L) \vdash G$

1	$C \vee S$	P

2	$C \to (G \land F)$	P
3	$S \to (G \land \neg L)$	P, G
4	$\mid \underline{C}$	A
5	$\mid G \land F$	$\to E$, 2,4
6	$\mid G$	$\land E$, 5
7	$\mid \underline{S}$	A
8	$\mid G \land \neg L$	$\to E$, 3,7
9	$\mid G$	$\land E$, 8
10	G	$\lor E$ 1, 4-6, 7-9

29. — Answer: $\vdash (R \to M) \lor \neg(P \land \neg P)$

1	$\mid \underline{(P \land \neg P)}$	A
2	$\mid P$	1 $\land E$
3	$\mid \neg P$	1 $\land E$
4	$\neg(P \land \neg P)$	1-3 $\neg I$
5	$(R \to M) \lor \neg(P \land \neg P)$	4$\lor I$

30. — Answer: $\vdash (P \land \neg S) \to ((\neg Q \lor S) \to \neg Q)$

1	$\mid \underline{P \land \neg S}$	A
2	$\mid \mid \underline{\neg Q \lor S}$	A
3	$\mid \mid \neg S$	1$\land E$
4	$\mid \mid \neg Q$	2,3DS
5	$\mid (\neg Q \lor S) \to \neg Q$	2-4$\to I$
6	$(P \land \neg S) \to ((\neg Q \lor S) \to \neg Q)$	1-5$\to I$