Directions: The following exam consists of 27 questions, for a total of 100 points. Read each question carefully (note: answers may break onto the next page). This exam tests your knowledge over the material from Chapter 3 and Chapter 4 of the course text and lectures. You may write on the test itself.

1 Definitions, Concepts, and Basic Mechanics

1. (2 points) What advantage does the truth table and truth tree tests have over the imagination test for validity?
   A. the truth table/tree tests are poetic; they take into account the spirit of human nature
   B. If an argument is persuasive in English, then the table/tree methods will tell us whether we ought to be persuaded by them.
   C. If an argument is deductively valid in English, then the truth table/tree method will always correctly determine whether it is (in fact) valid in the language of propositional logic (PL).
   D. The truth table/tree tests are mechanical (decision procedures)

2 Determining the truth of wffs

Directions: Determine the truth value of the wff. Note that in some cases you don't need to know all of the truth values for some (or all) of the propositional letters. Here are the following interpretations: $\mathscr{I}(A) = F$, $\mathscr{I}(B) = F$, $\mathscr{I}(C) = T$, $\mathscr{I}(D) = T$.

KEY: Select (A) for True and (B) for False.

2. (2 points) $v(\neg\neg A)$
3. (2 points) $v(\neg A \wedge B)$
4. (2 points) $v(A \wedge (B \vee C))$
5. (2 points) $v((A \wedge B) \wedge (C \wedge D))$
6. (2 points) $v(A \vee (B \wedge C))$
7. (2 points) $v(A \rightarrow \neg B)$
8. (2 points) $v(C \rightarrow (B \wedge \neg C))$
9. (2 points) $v(A \rightarrow (B \wedge \neg X))$
10. (2 points) $v(A \leftrightarrow (B \rightarrow A))$
11. (2 points) $v(\neg A \leftrightarrow \neg B)$
12. (2 points) $v(A \wedge \neg B) \vee \neg C$
13. (2 points) $v(X \vee \neg X)$

3 Truth-tree decomposition rules

Directions: Indicate the *first* decomposition rule that would be used to decompose the wffs below.

KEY: Let (A) $\neg\neg D$, (B) $\wedge D$, (C) $\neg \wedge D$, (D) $\vee D$, (E) $\neg \vee D$

14. (2 points) $\neg B \wedge \neg Q$?
15. (2 points) $\neg\neg A \vee \neg Z$
16. (2 points) $(Q \vee \neg L) \wedge M$
17. (2 points) $\neg\neg S$
18. (2 points) $\neg\neg(B \wedge Q)$
19. (2 points) $\neg(A \wedge R)$

KEY: Let (A) $\neg\neg D$, (B) $\rightarrow D$, (C) $\neg \rightarrow D$, (D) $\leftrightarrow D$, (E) $\neg \leftrightarrow D$

20. (2 points) $A \to \neg R$

21. (2 points) $\neg C \to \neg Z$

22. (2 points) $\neg A \leftrightarrow X$

23. (2 points) $\neg(A \to \neg R)$

24. (2 points) $\neg(B \leftrightarrow \neg\neg C)$

25. (2 points) $\neg\neg Z \leftrightarrow \neg M$?

4 Truth table and tree construction

Directions: On the answer sheet, construct a **truth table** that tests for the indicated property. To receive full credit, you must (1) construct the entire truth table (each row and each T and F), (2) label whether the table indicates the property in question (e.g., "tautology"), and (3) clearly explain why the table indicates the property in question (e.g., "The table shows ϕ is a tautology because ..."). Be sure (1) the table is fully complete (do not skip steps) and (2) Ts and Fs are clearly distinguishable (you can use 1 or O if it is easier).

26. (25 points) Determine whether $A \to B, \neg B \vdash \neg A$ is a valid sequent. A sequent is valid iff there is no interpretation where the premises are true and the conclusion is false. This is shown in the table below because there is no row where the premises are T and conclusion is F.

Directions: On the answer sheet, construct a **truth tree** that tests for the indicated property. To receive full credit, you must (1) construct the entire tree (numbering, the tree, and the node justification), (2) label whether the tree indicates the property in question (e.g., "tautology"), and (3) if an interpretation can be recovered from the tree, provide that interpretation.

27. (25 points) Determine whether $(B \to B) \land \neg(D \lor \neg Q)$ is a contingency, tautology, or contradiction.

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Solutions for pltablestrees/exam2sample.tex

1. D
2. B
3. B
4. B
5. B
6. B
7. A
8. B
9. A
10. B
11. A
12. B
13. A
14. $\wedge D$
15. $\vee D$
16. $\wedge D$
17. $\neg\neg D$
18. $\neg\neg D$
19. $\neg \wedge D$
20. $\rightarrow D$
21. $\rightarrow D$
22. $\leftrightarrow D$
23. $\neg \rightarrow D$
24. $\neg \leftrightarrow D$
25. $\leftrightarrow D$
26. $A \rightarrow B, \neg B \vdash \neg A$ is a valid sequent

A	B	$A \rightarrow B$	$\neg B$	$(A \rightarrow B) \wedge \neg B$	$\neg A$
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

27. Answer may vary, but the tree for this wff shows the wff is a contingency. It is a contingency since the tree test for contradiction shows that it is not a contradiction (all branches close), while the tree test for tautology shows that it is not a tautology (since the negated version of the wff yields a tree where all branches close). Since a wff is exactly one of the following (contingency, tautology, contradiction), the wff is a contingency.

Test for contradiction:

1.	$(B \to B) \wedge \neg(D \vee \neg Q)$ ✓	P
2.	$(B \to B)$ ✓	$1 \wedge D$
3.	$\neg(D \vee \neg Q)$	$1 \wedge D$
4.	$\neg D$	$3\neg \wedge D$
5.	$\neg Q$	$3\neg \wedge D$
6.	$\neg B$ B	$2 \to D$

Test for tautology:

1.	$\neg((B \to B) \wedge \neg(D \vee \neg Q))$ ✓		P
2.	$\neg(B \to B)$ ✓	$\neg\neg(D \vee \neg Q)$ ✓	$1 \wedge D$; $1\neg \wedge D$
3.	B	$D \vee \neg Q$ ✓	$2\neg \to D$; $2\neg\neg D$
4.	$\neg B$		$2\neg \to D$
5.		D $\neg Q$	$3 \vee D$

An interpretation may be recovered from the tree used in the contradiction test. One interpretation is $\mathscr{I}(B) = F, \mathscr{I}(Q) = F, \mathscr{I}(D) = F$