Directions: This exam has 34 questions, for a total of 100 points. Place your name on the answersheet (last page). Place proofs on the blank space on the answersheet.

1. (2 points) An interpretation of RL is a function that does what (indicate all that apply):
   A. specifies what objects are in the domain.
   B. assigns truth values to n-place predicate terms followed by n terms.
   C. for each name in RL it assigns that name one and only one item in $\mathcal{D}$
   D. for each n-place predicate term in RL assigns, it assigns that predicate term a set of n-tuples composed of elements from $\mathcal{D}$
2. (2 points) What is a derivation of ψ from Γ using **RD**?
   A. a finite string of formulas from a set Γ of RL wffs where (i) the last formula in the string is ψ and (ii) each formula is either a premise, an assumption, or is the result of the preceding formulas and the deductive apparatus.
   B. finite string of wffs starting with some premises Γ and ending with ψ.
   C. a finite string of wffs starting with some premises Γ or assumptions and ending with ψ.
   D. an infinite string of wffs starting with some premises Γ or assumptions and ending with ψ.
3. (2 points) What does the following mean: $\Gamma \vdash \mathbf{Q}$
   A. $\Gamma \vdash \mathbf{Q}$ says $\mathbf{Q}$ is a *syntactic* consequence of Γ (meaning that there is a derivation of $\mathbf{Q}$ from Γ).
   B. $\Gamma \vdash \mathbf{Q}$ says $\mathbf{Q}$ is a *semantic* consequence of Γ (there is no model such that the wffs of Γ are true and $\mathbf{Q}$ is false).
   C. $\Gamma \vdash \mathbf{Q}$ says $\mathbf{Q}$ is a *hypostatic abstraction* from Γ
   D. $\Gamma \vdash \mathbf{Q}$ says $\mathbf{Q}$ intuitively follows from Γ. That is, if you imagine Q in a proof, you can reason to Γ.
4. (2 points) Which of the following symbols are RL names (indicate all that apply)?
   A. $\forall$
   B. b
   C. c
   D. x

Directions: Write any free variables in the following wffs on the line. If there are no free variables, write "none".

5. (2 points) Px
6. (2 points) Gab
7. (2 points) $(\forall x)(\forall y)Gyx$

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, $\mathscr{I}(e) = 5$, for all other names α, $\mathscr{I}(\alpha) = 4$, $\mathscr{I}(N) = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(G) = \{\langle 2, 1\rangle, \langle 3, 2\rangle, \langle 3, 1\rangle, \langle 5, 1\rangle\}$, $\mathscr{I}(I) = \{\}$, $\mathscr{I}(E) = \{2, 4\}$, $\mathscr{I}(O) = \{1, 3, 5\}$

8. (2 points) Ea
9. (2 points) Od
10. (2 points) $(\forall x)Ex$
11. (2 points) $\neg(\exists y)Oy$
12. (2 points) $(\exists x)(Nx \wedge \neg Ox)$

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{Tek, Shinji, Lain\}$, $\mathscr{I}(t) = Tek$, $\mathscr{I}(s) = Shinji$, $\mathscr{I}(l) = Lain$, for all other names α, $\mathscr{I}(\alpha) = Shinji$, $\mathscr{I}(Lxy) = \{\langle Tek, Tek\rangle, \langle Tek, Lain\rangle, \langle Lain, Lain\rangle\}$, $\mathscr{I}(Hx) = \{Tek\}$

13. (2 points) $\neg Hs$
14. (2 points) Llt
15. (2 points) $(\forall x)\neg(Lxs)$
16. (2 points) $(\forall x)\neg(Lsx)$
17. (2 points) $(\exists x)Hx$
18. (2 points) $(\forall x)(Hx \rightarrow Lxx)$

Directions: Translate the wffs below into English using the following key: $\mathcal{D} = \{Tek, Shinji, Lain\}$, $t = Tek$, $s = Shinji$, $l = Lain$, for all other names α, $\alpha = Shinji$, $Lxy = x$ loves y, $Hx = x$ is happy. $Rx = x$ is rich.

19. (2 points) $\neg Hs$
20. (2 points) Llt
21. (2 points) $(\forall x)Lxl$
22. (2 points) $\neg(\exists x)Lxs$
23. (2 points) $(\exists x)(Hx \wedge Lxx)$
24. (2 points) $(\forall x)(Rx \rightarrow Lxx)$

Directions: Writing the abbreviation for the single derivation rule that is represented in the following:

25. (2 points) $(\forall z)(Qz \rightarrow Qz) \vdash Qc \rightarrow Qc$
26. (2 points) $Ca \wedge Ca \vdash (\exists z)(Cz \wedge Cz)$
27. (2 points) $\neg Qab \vdash (\exists z)\neg Qxb$
28. (2 points) From Bb to $(\forall y)By$ provided (1) b is not in a premise or in an assumption of an active subproof and (2) b is not in $(\forall y)By$?
29. (2 points) Starting from $(\exists x)Fx$, suppose Fa is assumed. Next, suppose χ is derived in the subproof starting with Fa. Finally, suppose χ is deprived using $(\exists x)Fx$ and the entire subproof.
30. (2 points) $\neg(\forall z)Zz \vdash (\exists z)\neg Zz$

Directions: Provide proofs for the following:

31. (10 points) $Sb, (\forall x)Qx \vdash (\exists x)Qx$
32. (10 points) $\neg Fbb \wedge Fba \vdash (\exists x)(\exists y)\neg Fxy$
33. (10 points) $Qa \wedge Pa, (\forall x)(Px \rightarrow Mx), (\forall x)Px \vdash (\forall x)Mx$
34. (10 points) $(\exists x)\neg Wx \vdash (\exists x)(Mx \vee \neg Wx)$

Solutions for exam4/exam4kA

1. C, (D)
2. A
3. A
4. B, (C)
5. x
6. none
7. none
8. F
9. F
10. F
11. F
12. T
13. T
14. F
15. T
16. T
17. T
18. T
19. Shinji is not happy.
20. Lain loves Tek.
21. Everyone loves Lain.
22. No one loves Shinji. Also: It is not the case that someone exists that loves Shinji.
23. Someone who is happy loves themselves.
24. All rich people love themselves.
25. $\forall E$
26. $\exists I$
27. $\exists I$
28. $\forall I$
29. $\exists E$
30. QN
31. $Sb, (\forall x)Qx \vdash (\exists y)Qy$

1	Sb	P
2	$(\forall x)Qx$	P, $(\exists y)Qy$
3	Qa	2, $\forall E$
4	$(\exists y)Qy$	3, $\exists I$

32. $\neg Fbb \wedge Fba \vdash (\exists x)(\exists y)\neg Fxy$

1	$\neg Fbb \wedge Fba$	P, $(\exists x)(\exists y)\neg Fxy$
2	$\neg Fbb$	1 $\wedge E$
3	$(\exists y)\neg Fby$	2, $\exists I$
4	$(\exists x)(\exists y)\neg Fxy$	3, $\exists I$

33. $Qa \wedge Pa, (\forall x)(Px \rightarrow Mx), (\forall x)Px \vdash (\forall x)Mx$

1	$Qa \wedge Pa$	P
2	$(\forall x)(Px \rightarrow Mx)$	P
3	$(\forall x)Px$	P, $(\forall x)Mx$
4	$Pb \rightarrow Mb$	2 $\forall E$
5	Pb	3 $\forall E$

6	Mb	4,5 $\rightarrow E$
7	$(\forall x)Mx$	6 $\forall I$

34. $(\exists x)\neg Wx \vdash (\exists x)(Mx \vee \neg Wx)$

1	$(\exists x)\neg Wx$	P, $(\exists x)(Mx \vee \neg Wx)$
2	$\vert\ \overline{\neg Wa}$	A
3	$\vert\ Ma \vee \neg Wa$	2 $\vee I$
4	$\vert\ (\exists x)(Mx \vee \neg Wx)$	3 $\exists I$
5	$(\exists x)(Mx \vee \neg Wx)$	1, 2-4 $\exists E$