Directions: This exam has 42 questions, for a total of 100 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

0.1 Conceptual Questions and Definitions

Choose the best answer.

1. (2 points) An interpretation of **RL** is a function that does what (indicate all that apply):
   - A. specifies what objects are in the domain.
   - B. assigns truth values to n-place predicate terms followed by n terms.
   - C. for each name in **RL** it assigns that name one and only one item in $\mathcal{D}$
   - D. for each n-place predicate term in **RL**assigns, it assigns that predicate term a set of n-tuples composed of elements from $\mathcal{D}$
   - E. assigns truth values to objects and wffs
2. (2 points) What is the principal weakness of **PL** in comparison to **RL**
   - A. **PL** is not expressive enough: there are valid English arguments that can be expressed in **RL** that cannot be expressed in **PL**
   - B. **PL** is too expressive: there are valid arguments in **PL** for which it would be impossible to express in English.
   - C. **PL** has an imprecise syntax, while the syntax of **RL** is fully precise.
   - D. **PL** has an imprecise semantics, while the semantics of **RL** is fully precise.
3. (2 points) What does it mean to say that a variable is bound?
   - A. A variable is bound if and only if it is in the scope of a quantifier that quantifies for *some* variable.
   - B. A variable is bound if and only if it is a name.
   - C. A variable is bound if and only if it is in the scope of a quantifier.
   - D. A variable is bound if and only if it is in the scope of a quantifier that quantifies for that variable.
4. (2 points) What determines whether a formula is an RL-wff?
   - A. the semantics of RL
   - B. the number of names in RL
   - C. the truth values of the various wffs, including the quantified wffs
   - D. the RL-formation rules
5. (2 points) What is a model ($\mathcal{D}$)?
   - A. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and an interpretation function ($\mathscr{I}$)
   - B. a model ($\mathcal{M}$) is a three-part structure consisting of a domain ($\mathcal{D}$), an interpretation function ($\mathscr{I}$), and a valuation (v) function.
   - C. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and a valuation function v where the valuation function assigns truth values to RL-wffs.
   - D. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$)

6. (2 points) What does a valuation function do in predicate logic? Indicate the best answer.
    A. Takes a well-formed formula and assigns it a truth value (T or F) relative to a model.
    B. tells you whether a wff is properly decomposed
    C. it is a function that takes a name and assigns it to an object in a model.
    D. it is a function that takes an n-place predicate term and assigns it a set of n-tuples.

0.2 Symbols

7. (2 points) Which of the following symbols are **RL** names (indicate all that apply)?
    A. b
    B. y
    C. $\exists$
    D. m
    E. n
    F. $\forall$
    G. $\diamond$

0.3 Syntax

Identify any free variables in the following wffs by writing the free variable on the line. If there are no free variables, write "none".

8. (2 points) Rxy
9. (2 points) Rab
10. (2 points) $(\forall y)Wxy$
11. (2 points) $(\forall x)(\forall y)Pyx$

State whether the following formulas are wffs. You can assume that H is a one-place predicate, that L is a two-place predicate, and conventions for simplifying wffs are present.

12. (2 points) Hab
13. (2 points) $Ha \vee \neg Hb$
14. (2 points) $(\forall x)Hx$
15. (2 points) $\neg(\exists x)(\exists y)\neg Lxy$
16. (2 points) $(\exists y)(\forall x)(Lxy \wedge \neg Hx)$

0.4 Semantics

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, $\mathscr{I}(e) = 5$, for all other names α, $\mathscr{I}(\alpha) = 4$, $\mathscr{I}(N) = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(G) = \{\langle 2, 1\rangle, \langle 3, 2\rangle, \langle 3, 1\rangle, \langle 5, 1\rangle\}$, $\mathscr{I}(I) = \{\}$, $\mathscr{I}(E) = \{2, 4\}$, $\mathscr{I}(O) = \{1, 3, 5\}$

17. (2 points) $\neg Eb$
18. (2 points) Oc
19. (2 points) $(\forall x)Nx$
20. (2 points) $(\exists y)Iy$
21. (2 points) $(\forall x)(Ex \wedge Ox)$
22. (2 points) Gab
23. (2 points) $(\exists x)(Gex)$

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{Jon, Tek, Liz\}$, $\mathscr{I}(a) = Jon$, $\mathscr{I}(b) = Tek$, $\mathscr{I}(c) = Liz$, for all other names α, $\mathscr{I}(\alpha) = Liz$, $\mathscr{I}(Lxy) = \{\langle Jon, Liz\rangle, \langle Tek, Liz\rangle, \langle Liz, Liz\rangle\}$, $\mathscr{I}(Hx) = \{Jon, Liz\}$

24. (2 points) Lba
25. (2 points) Lcc
26. (2 points) $\neg Laa$
27. (2 points) $(\exists x)Lax$
28. (2 points) $(\exists x)Lxx$
29. (2 points) $(\forall x)(Hx \rightarrow Lxx)$
30. (2 points) $(\exists x)(Hx \wedge \neg Lxx)$

0.5 Translation

Directions: Translate the following English sentences into the language of predicate logic. Write the formula on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a) =$ Ava, $\mathscr{I}(j) =$ Jon, $\mathscr{I}(e) =$ Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

31. (2 points) Ava is not happy
32. (2 points) Ava loves Jon.
33. (2 points) Someone is *both* rich and happy.
34. (2 points) Someone is rich and someone is happy.
35. (2 points) All happy people are rich.

Directions: Translate the following predicate logic wffs into English. Write your translation on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a) =$ Ava, $\mathscr{I}(j) =$ Jon, $\mathscr{I}(e) =$ Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

36. (2 points) $(\forall x)Lax$

37. (2 points) $(\forall x)Lxa$

38. (2 points) $(\exists x)(Rx \wedge Ljx)$

39. (2 points) $(\forall x)(Lxx \rightarrow Hx)$

40. (2 points) $(\exists x)(Rx \wedge \neg Lxx)$

0.6 Truth trees and models

Directions: Use a truth-tree to determine whether the following sets of wffs are consistent/inconsistent or arguments are valid/invalid. If the tree shows the set to be consistent or the argument to be invalid, construct a model illustrating this fact. (Rubric: Tree=5pts, Property=1pt, Model=4pts, if applicable)

41. (10 points) Determine consistent/inconsistent: $Pa, Qb, (\exists x)\neg Px, (\forall x)Qx$

42. (10 points) Determine validity/invalidity: $(\exists x)(Px \wedge Rx), \neg(\forall x)Lxx \models \neg Laa$

0.7 Bonus Questions

$P \wedge Q$ $P \quad \wedge D$ $Q \quad \wedge D$	$P \vee Q$ $P \quad Q \quad \vee D$
$\neg(P \vee Q)$ $\neg(P) \quad \neg \vee D$ $\neg(Q) \quad \neg \vee D$	$\neg(P \wedge Q)$ $\neg(P) \quad \neg(Q) \quad \neg \wedge D$
$\neg(P \rightarrow Q)$ $P \quad \neg \rightarrow D$ $\neg(Q) \quad \neg \rightarrow D$	$(P \rightarrow Q)$ $\neg(P) \quad Q \quad \rightarrow D$
$P \leftrightarrow Q$ $P \quad \neg(P) \quad \leftrightarrow D$ $Q \quad \neg(Q) \quad \leftrightarrow D$	$\neg\neg(P)$ $P \quad \neg\neg D$
$\neg(P \leftrightarrow Q)$ $P \quad \neg(P) \quad \neg \leftrightarrow D$ $\neg(Q) \quad Q \quad \neg \leftrightarrow D$	
$\neg(\exists x)\phi\checkmark$ $(\forall x)\neg(\phi), \neg\exists D$	$\neg(\forall x)\phi\checkmark$ $(\exists x)\neg(\phi), \neg\forall D$
$(\exists x)\phi\checkmark$ $\phi(\alpha/x), \exists D$	$(\forall x)\phi$ $\phi(\alpha/x), \forall D$

Table 1: Truth tree decomposition rules for **PL** and **RL**

Directions: Please write your **name** on the top of this page. Answer all of the questions on the answer sheet provided. If an answer will not fit on the blank provided, place your answer on one of the several blank pages.

1. _____
2. _____
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