Directions: This exam has 34 questions, for a total of 100 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. Please place your name on the answer sheet.

1. (2 points) What is a model ($\mathcal{D}$)?
   A. A MODEL ($\mathcal{M}$) IS A TWO-PART STRUCTURE CONSISTING OF A DOMAIN ($\mathcal{D}$) AND AN INTERPRETATION FUNCTION ($\mathscr{I}$)
   B. a model ($\mathcal{M}$) is a three-part structure consisting of a domain ($\mathcal{D}$), an interpretation function ($\mathscr{I}$), and a valuation (v) function.
   C. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and a valuation function v where the valuation function assigns truth values to RL-wffs.
   D. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$)

2. (2 points) What is a derivation of **Q** using **RD**?
   A. A FINITE STRING OF FORMULAS FROM A SET Γ OF **RL** WFFS WHERE (I) THE LAST FORMULA IN THE STRING IS Q AND (II) EACH FORMULA IS EITHER A PREMISE, AN ASSUMPTION, OR IS THE RESULT OF THE PRECEDING FORMULAS AND THE DEDUCTIVE APPARATUS.
   B. finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with **Q**.
   C. a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.
   D. an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.

3. (2 points) What does the following mean: $\Gamma \vdash \mathbf{Q}$
   A. $\Gamma \vdash \mathbf{Q}$ SAYS **Q** IS A *syntactic* CONSEQUENCE OF Γ (MEANING THAT THERE IS A DERIVATION OF **Q** FROM Γ).
   B. $\Gamma \vdash \mathbf{Q}$ says **Q** is a *semantic* consequence of Γ (there is no model such that the wffs of Γ are true and **Q** is false).
   C. $\Gamma \vdash \mathbf{Q}$ says **Q** is a *hypostatic abstraction* from Γ
   D. $\Gamma \vdash \mathbf{Q}$ says **Q** intuitively follows from Γ. That is, if you imagine Q in a proof, you can reason to Γ.

4. (2 points) Which of the following symbols are **RL** names (indicate all that apply)?
   A. b
   B. m
   C. n
   D. $\forall$
   E. $\diamond$

Directions: Identify any free variables in the following wffs by writing the free variable on the line. If there are no free variables, write "none".

5. (2 points) Rxy

   5. XY

6. (2 points) Rab

   6. NONE

7. (2 points) $(\forall x)(\forall y)Pyx$

7. NONE

> **Directions:** Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, $\mathscr{I}(e) = 5$, for all other names α, $\mathscr{I}(\alpha) = 4$, $\mathscr{I}(N) = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(G) = \{\langle 2, 1\rangle, \langle 3, 2\rangle, \langle 3, 1\rangle, \langle 5, 1\rangle\}$, $\mathscr{I}(I) = \{\}$, $\mathscr{I}(E) = \{2, 4\}$, $\mathscr{I}(O) = \{1, 3, 5\}$

8. (2 points) $\neg Eb$

8. F

9. (2 points) Oc

9. T

10. (2 points) $(\forall x)Nx$

10. F

11. (2 points) $(\exists y)Iy$

11. F

12. (2 points) $(\forall x)(Ex \wedge Ox)$

12. F

> **Directions:** Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{Jon, Tek, Liz\}$, $\mathscr{I}(a) = Jon$, $\mathscr{I}(b) = Tek$, $\mathscr{I}(c) = Liz$, for all other names α, $\mathscr{I}(\alpha) = Liz$, $\mathscr{I}(Lxy) = \{\langle Jon, Liz\rangle, \langle Tek, Liz\rangle, \langle Liz, Liz\rangle\}$, $\mathscr{I}(Hx) = \{Jon, Liz\}$

13. (2 points) Lba

13. F

14. (2 points) $\neg Laa$

14. T

15. (2 points) $(\exists x)Lax$

15. T

16. (2 points) $(\exists x)Lxx$

16. T

17. (2 points) $(\forall x)(Hx \rightarrow Lxx)$

17. F

18. (2 points) $(\exists x)(Hx \wedge \neg Lxx)$

18. T

Directions: Translate the following predicate logic wffs into English. Write your translation on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a)$ = Ava, $\mathscr{I}(j)$ = Jon, $\mathscr{I}(e)$ = Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

19. (2 points) Ha

Solution: Ava is happy

20. (2 points) Lea

Solution: Eve loves Ava

21. (2 points) $(\forall x)Lxx$

Solution: Everyone loves themselves.

22. (2 points) $(\forall x)Lxa$

Solution: Everyone loves Ava.

23. (2 points) $(\exists x)(Rx \wedge Ljx)$

Solution: Someone who is rich is loved by Jon.

24. (2 points) $(\forall x)(Hx \rightarrow Lxx)$

Solution: All happy people love themselves.

Directions: Answer the questions on the line provided by writing the abbreviation for the derivation rule that is best described in the question prompt provided.

25. (2 points) What derivation rule is best described as follows: from the existentially quantified wff $(\exists x)\phi(x)$ and a subproof that begins with an assumption $\phi(\alpha/x)$, a wff ψ may be derived provided (i) the name α is foreign to the proof and (ii) ψ does not contain α.

25. $\exists E$

26. (2 points) What derivation rule is best described as follows: given a wff Za, a universally quantified wff can be derived (e.g., $(\forall x)Zx$) provided (i) a does not occur as a premise or as an assumption in an open subproof, and (2) a does not occur in $(\forall x)Zx$.

26. $\forall I$

27. (2 points) What single derivation rule would allow you to reason to Lbb from $(\forall x)Lxx$?

27. $\forall E$

28. (2 points) What single derivation rule would allow you to reason to $(\exists x)Lxx$ from Laa?

28. $\exists I$

29. (2 points) What single derivation rule would allow you to reason to $(\forall x)\neg(Px \vee Qx)$ from $\neg(\exists x)(Px \vee Qx)$?

29. QN

30. (2 points) Assuming "a" is not present in an active assumption nor a premise, what single derivation rule would allow you to derive $(\forall x)(Lx \to Mx)$ from $La \to Ma$?

30. $\forall I$

Directions: Provide proofs for the following.

31. (10 points) $(\forall x)Px \vdash (\exists x)Px$

Solution: Hint: try using $\forall E$ then $\exists I$

32. (10 points) $Lab \vdash (\exists x)(\exists y)Lxy$

Solution: Use of $\exists I$ twice.

33. (10 points) $Pa, Qb, (\forall x)Px \wedge (\forall y)Qy \vdash (\forall x)(Px \wedge Qx)$

Solution: Tests the multiple uses of $\forall I$. To start, try using $\wedge E$ and then use $\forall E$.

34. (10 points) $(\exists x)Mx \vdash (\exists x)(Mx \vee \neg Qx)$

Solution: Take a look at the example uses of $\exists E$ in the textbook or from classnotes as this proof is there. But also, hint: try using $\exists E$.

Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$
$P, Q \vdash Q \wedge P$

Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Rule – Conditional Introduction ($\to I$)

n	$\mid P$	A
$\vdots$	$\mid \vdots$	
$(n+1)$	$\mid Q$	
$(n+2)$	$P \to Q$	$\to I$, n–$(n+1)$

Rule – Conditional Elimination ($\to E$)

$P \to Q, P \vdash Q$

Rule – Reiteration (R)

$P \vdash P$

Rule – Negation Introduction ($\neg I$)

n	$\mid P$	A
$\vdots$	$\mid \vdots$	
$(n+1)$	$\mid Q$	
$(n+2)$	$\mid \neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Rule – Negation Elimination ($\neg E$)

$$
\begin{array}{lll}
n & \quad\vert\ \underline{\neg(P)} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ Q & \\
(n+2) & \quad\vert\ \neg Q & \\
(n+3) & P & \neg E,\ n\text{–}(n+2)
\end{array}
$$

Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Rule – Disjunction Elimination ($\vee E$)

$$
\begin{array}{lll}
1 & P \vee Q & \text{P} \\
n & \quad\vert\ \underline{P} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ R & \\
(i) & \quad\vert\ \underline{Q} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(i+1) & \quad\vert\ R & \\
(k) & R & \vee E,\ 1,\ n\text{–}(n+1),\ (i)\text{–}(i+1)
\end{array}
$$

Rule – Biconditional Introduction ($\leftrightarrow I$)

$$
\begin{array}{lll}
n & \quad\vert\ \underline{P} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ Q & \\
(i) & \quad\vert\ \underline{Q} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(i+1) & \quad\vert\ P & \\
(k) & P \leftrightarrow Q & \leftrightarrow I,\ n\text{–}(n+1),\ (i)\text{–}(i+1)
\end{array}
$$

Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Rule – Disjunctive Syllogism (DS)

$P \vee Q, \neg Q \vdash P$ or $P \vee Q, \neg P \vdash Q$

Rule – Modus Tollens (MT)

$P \rightarrow Q, \neg Q \vdash \neg P$

Rule – Hypothetical Syllogism (HS)

$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Rule – De Morgan's Laws (DeM)

$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$

Rule – Implication (IMP)

$P \rightarrow Q \dashv\vdash \neg P \vee Q$

Rule – Universal Elimination ($\forall E$)

$(\forall x)\phi(x_1 \ldots x_n) \vdash \phi(\alpha_1 \ldots \alpha_n / x_1 \ldots x_n)$ where x is not in $\phi(\alpha_1 \ldots \alpha_n)$

Rule – Existential Introduction ($\exists I$)

$\phi(\alpha_i) \vdash (\exists x)\phi(x_n/\alpha_n)$ where x is not in $\phi(\alpha_i)$

Rule – Universal Introduction ($\forall I$)

$\phi(\alpha_1 \ldots \alpha_n) \vdash (\forall x)\phi(x_1 \ldots x_n/\alpha_1, \ldots \alpha_n)$ where the name α does not occur as premise, as an assumption in an open subproof, or in $(\forall x)\phi(x_1 \ldots x_n/\alpha_1, \ldots \alpha_n)$ and where x is not in $\phi(\alpha_1 \ldots \alpha_n)$

Rule – Existential Elimination ($\exists E$)

$$
\begin{array}{lll}
1 & (\exists x)\mathbf{P} & \text{P} \\
n & \quad\vert\ \underline{\mathbf{P}(a/x)} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ \mathbf{Q} & \\
(k) & \mathbf{Q} & \exists E,\ 1,\ n\text{–}(n+1)
\end{array}
$$

Rule – Quantifier Negation (QN)

$\neg(\forall x)\phi \dashv\vdash (\exists x)\neg\phi$
$\neg(\exists x)\phi \dashv\vdash (\forall x)\neg\phi$

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30. ______________________________