Directions: This exam has 42 questions, for a total of 100 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

0.1 Conceptual Questions and Definitions

Choose the best answer.

1. (2 points) An interpretation of **RL** is a function that does what (indicate all that apply):
   A. specifies what objects are in the domain.
   B. assigns truth values to n-place predicate terms followed by n terms.
   C. assigns truth values to objects in the domain
   D. for each name in **RL** it assigns that name one and only one item in $\mathcal{D}$
   E. for each n-place predicate term in **RL** assigns, it assigns that predicate term a set of n-tuples composed of elements from $\mathcal{D}$

2. (2 points) What is the principal weakness of **PL** in comparison to **RL**
   A. **PL** is not expressive enough: there are valid English arguments that can be expressed in **RL** that cannot be expressed in **PL**
   B. **PL** is too expressive: there are valid arguments in **PL** for which it would be impossible to express in English.
   C. **PL** has an imprecise syntax, while the syntax of **RL** is fully precise.
   D. **PL** has an imprecise semantics, while the semantics of **RL** is fully precise.

3. (2 points) What does it mean to say that a variable is bound?
   A. A variable is bound if and only if it is in the scope of a quantifier that quantifies for *some* variable.
   B. A variable is bound if and only if it is a name.
   C. A variable is bound if and only if it is in the scope of a quantifier.
   D. A variable is bound if and only if it is in the scope of a quantifier that quantifies for that variable.

4. (2 points) What is a model ($\mathcal{D}$)?
   A. a model ($\mathcal{M}$) is a three-part structure consisting of a domain ($\mathcal{D}$), an interpretation function ($\mathscr{I}$), and a valuation (v) function.
   B. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$).
   C. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and an interpretation function ($\mathscr{I}$)
   D. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$) and a valuation (v) function.

5. (2 points) What are the ways to specify the items of a domain?
   A. the items of a domain ($\mathcal{D}$) can *only* be specified either by listing each of the items individually.
   B. the items of a domain ($\mathcal{D}$) can *only* be specified by indicating what property all of the members of the domain have.
   C. the items of a domain ($\mathcal{D}$) are never specified.
   D. the items of a domain ($\mathcal{D}$) can be specified either by listing each of the items individually or by indicating what property all of the members of the domain have.

6. (2 points) What two things does a predicate logic interpretation do?
   A. takes predicate logic names as input and gives items from the domain as output (viz., interprets names)
   B. takes predicate logic n-place predicate terms as input and gives sets of items from the domain as output (viz., interprets predicates)
   C. takes quantifiers as input and delivers quantification as output
   D. takes truth-functional operators as input and delivers truth values as output
   E. takes parentheses as input and delivers parentheses as output

0.2 Symbols

7. (2 points) Which of the following symbols are **RL** names (indicate all that apply)?
   A. c
   B. d
   C. x
   D. y
   E. $\exists$
   F. $\forall$

0.3 Syntax

Identify any free variables in the following wffs (if there are no free variables, write "none"):

8. (2 points) $(\exists y)Lyy$
9. (2 points) $(\forall x)(\forall y)Pyx$
10. (2 points) Pa
11. (2 points) $(\forall x)(\exists y)(Rxy \rightarrow Pa)$

State whether the following formulas are wffs. You can assume that P is a one-place predicate, that L is a two-place predicate, and conventions for simplifying wffs are present.

12. (2 points) Pa
13. (2 points) $(\forall x)Px$
14. (2 points) $(\forall x)\neg Px$
15. (2 points) $(\exists x)(\forall z)Lxz$
16. (2 points) $(\forall x)\neg(\exists y)Lxy$

0.4 Semantics

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, $\mathscr{I}(e) = 5$, $\mathscr{I}(N) = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(E) = \{2, 4\}$, $\mathscr{I}(O) = \{1, 3, 5\}$, $\mathscr{I}(G) = \{\langle 2, 1\rangle, \langle 3, 2\rangle\}$

17. (2 points) Ob
18. (2 points) $(\exists x)Nx$
19. (2 points) $(\forall x)Ex$
20. (2 points) $(\forall x)(Ox \rightarrow Ex)$
21. (2 points) $(\exists x)Gxa$
22. (2 points) $(\forall x)(Gxx \rightarrow Ex)$

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{Judit, Tek, Liz\}$, $\mathscr{I}(l) = Liz$, $\mathscr{I}(t) = Tek$, $\mathscr{I}(j) = Judit$, $\mathscr{I}(Lxy) = \{\langle Judit, Liz\rangle, \langle Liz, Judit\rangle, \langle Liz, Liz\rangle\}$, $\mathscr{I}(Hx) = \{Tek, Judit, Liz\}$

23. (2 points) Lll
24. (2 points) $(\exists x)Ljx$
25. (2 points) $(\forall x)Lxx$
26. (2 points) $(\exists x)(Lxx \wedge Hx)$
27. (2 points) $(\forall x)(Lxx \rightarrow Hx)$

0.5 Translation

Directions: Translate the following English sentences into the language of predicate logic or predicate logic wff into English. Write the formula or sentence on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a)$ = Ava, $\mathscr{I}(j)$ = Jon, $\mathscr{I}(e)$ = Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

28. (2 points) Ava is happy
29. (2 points) Ava loves herself.
30. (2 points) Someone is *both* rich and happy.
31. (2 points) Someone is rich and someone is happy.
32. (2 points) All rich people are happy.
33. (2 points) $(\exists x)\neg Rx$
34. (2 points) $(\forall x)Lxa$
35. (2 points) $(\forall x)\neg Lax$
36. (2 points) $(\forall x)(Lxx \rightarrow Hx)$
37. (2 points) $(\exists x)(Rx \wedge \neg Lxx)$

0.6 Truth tree decomposition

38. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $(\exists x)Px$

39. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $(\exists x)\neg Px$

40. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg(\exists x)\neg Px$

0.7 Truth trees and models

Directions: Use a truth-tree to determine whether the following sets of wffs are consistent/inconsistent or arguments are valid/invalid. If the tree shows the set to be consistent or the argument to be invalid, construct a model illustrating this fact. (Rubric: Tree=5pts, Property=1pt, Model=4pts, if applicable)

41. (10 points) Determine consistent/inconsistent: $Qb, Pa, (\exists x)\neg Qx, (\forall y)Py$

42. (10 points) Determine valid/invalid: $(\exists x)Px, (\exists x)Lxx \models Laa$

$P \wedge Q$ $P \quad \wedge D$ $Q \quad \wedge D$	$P \vee Q$ $P \qquad Q \quad \vee D$
$\neg(P \vee Q)$ $\neg(P) \quad \neg \vee D$ $\neg(Q) \quad \neg \vee D$	$\neg(P \wedge Q)$ $\neg(P) \qquad \neg(Q) \quad \neg \wedge D$
$\neg(P \to Q)$ $P \quad \neg \to D$ $\neg(Q) \quad \neg \to D$	$P \leftrightarrow Q$ $P \qquad \neg(P) \quad \leftrightarrow D$ $Q \qquad \neg(Q) \quad \leftrightarrow D$
$\neg\neg(P)$ $P \quad \neg\neg D$	$\neg(P \leftrightarrow Q)$ $P \qquad \neg(P) \quad \neg \leftrightarrow D$ $\neg(Q) \qquad Q \quad \neg \leftrightarrow D$
$\neg(\exists x)\phi\checkmark$ $(\forall x)\neg(\phi), \neg\exists D$	$\neg(\forall x)\phi\checkmark$ $(\exists x)\neg(\phi), \neg\forall D$
$(\exists x)\phi\checkmark$ $\phi(\alpha/x), \exists D$	$(\forall x)\phi$ $\phi(\alpha/x), \forall D$

Table 1: Truth tree decomposition rules for **PL** and **RL**

Directions: Please write your **name** on the top of this page. Answer all of the questions on the answer sheet provided. If an answer will not fit on the blank provided, place your answer on one of the several blank pages.

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________
9. __________
10. __________
11. __________
12. __________
13. __________
14. __________
15. __________
16. __________
17. __________
18. __________
19. __________
20. __________
21. __________
22. __________
23. __________
24. __________
25. __________
26. __________
27. __________
28. __________
29. __________
30. __________
31. __________
32. __________
33. __________
34. __________
35. __________
36. __________
37. __________
38. __________
39. __________
40. __________