Directions: This exam has 42 questions, for a total of 0 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

0.1 Conceptual Questions and Definitions

Choose the best answer.

1. (2 points) What is the principal weakness of **PL** in comparison to **RL**
   A. **PL** is too expressive: there are valid arguments in **PL** for which it would be impossible to express in English.
   B. **PL** has an imprecise syntax, while the syntax of **RL** is fully precise.
   C. **PL** has an imprecise semantics, while the semantics of **RL** is fully precise.
   D. **PL** IS NOT EXPRESSIVE ENOUGH: THERE ARE VALID ENGLISH ARGUMENTS THAT CAN BE EXPRESSED IN **RL** THAT CANNOT BE EXPRESSED IN **PL**

2. (2 points) An interpretation of **RL** is a function that does what (indicate all that apply):
   A. specifies what objects are in the domain.
   B. assigns truth values to propositional letters
   C. assigns truth values to n-place predicate terms followed by n terms.
   D. ASSIGNS OBJECTS IN $\mathcal{D}$ TO EACH NAME IN **RL**
   E. ASSIGNS A SET OR COLLECTION OF OBJECTS IN $\mathcal{D}$ TO n-PLACE PREDICATE TERMS IN **RL**

3. (2 points) What is a bound variable?
   A. A variable is bound if and only if it is in the scope of a quantifier that quantifies for some variable.
   B. A variable is bound if and only if it is a name.
   C. A variable is bound if and only if it is in the scope of a quantifier.
   D. A VARIABLE IS BOUND IF AND ONLY IF IT IS IN THE SCOPE OF A QUANTIFIER THAT QUANTIFIES FOR THAT VARIABLE.

4. (2 points) What is a model ($\mathcal{D}$)?
   A. a model ($\mathcal{M}$) is a three-part structure consisting of a domain ($\mathcal{D}$), an interpretation function ($\mathscr{I}$), and a valuation (v) function.
   B. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$).
   C. A MODEL ($\mathcal{M}$) IS A TWO-PART STRUCTURE CONSISTING OF A DOMAIN ($\mathcal{D}$) AND AN INTERPRETATION FUNCTION ($\mathscr{I}$)
   D. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$) and a valuation (v) function.

5. (2 points) What are the ways to specify the items of a domain?
   A. the items of a domain ($\mathcal{D}$) can *only* be specified either by listing each of the items individually.
   B. the items of a domain ($\mathcal{D}$) can *only* be specified by indicating what property all of the members of the domain have.
   C. the items of a domain ($\mathcal{D}$) are never specified.
   D. THE ITEMS OF A DOMAIN ($\mathcal{D}$) CAN BE SPECIFIED EITHER BY LISTING EACH OF THE ITEMS INDIVIDUALLY OR BY INDICATING WHAT PROPERTY ALL OF THE MEMBERS OF THE DOMAIN HAVE.

6. (2 points) In a predicate logic tree, under what conditions is a branch that contains a universally quantified wff (e.g. $(\forall x)Px$) considered a *completed open branch* (indicate all that apply)

   A. when $(\forall x)Px$ has been decomposed into $\neg(\exists x)Px$

   B. when $(\forall x)Px$ has been decomposed for at least one name $a, b, c, \ldots$ that occurs in that branch

   C. WHEN $(\forall x)Px$ HAS BEEN DECOMPOSED FOR EVERY NAME $a, b, c, \ldots$ THAT OCCURS IN THAT BRANCH

   D. WHEN ALL THE COMPLEX WFFS (NON-LITERALS) THAT ARE IN THAT BRANCH AND THAT CAN BE DECOMPOSED HAVE BEEN DECOMPOSED

   E. WHEN THE BRANCH IS NOT CLOSED, VIZ., DOES NOT CONTAIN A WFF AND ITS LITERAL NEGATION

0.2 Symbols

7. (2 points) Which of the following symbols are **RL** names (indicate all that apply)?

   A. a

   B. y

   C. x

   D. b

   E. d

   F. $\forall$

0.3 Syntax

Identify any free variables in the following wffs (if there are none, indicate "none"):

8. (2 points) $(\exists y)Rxy$

9. (2 points) $(\forall x)Pyx$

10. (2 points) $(\forall y)(\forall x)Ryb$

11. (2 points) $(\forall x)(\exists y)(Rxy \rightarrow Pa)$

Which of the following wffs below are **closed** wffs in **RL** Let Px be a one-place predicate and Rxy be a two-place predicate.

12. (2 points) $Rac, (\exists x)(\forall y)Rxy, (\forall x)(Rxa \rightarrow Py), (\forall x)Rxx$

State whether the following formulas are wffs. You can assume that H is a one-place predicate, that L is a two-place predicate, and conventions for simplifying wffs are present.

13. (2 points) $(\exists x)Hxx$

13. NO

14. (2 points) Hd

14. YES

15. (2 points) $(\exists x)\neg Lxa$

15. YES

16. (2 points) $\neg(\exists x)\neg Lx$

16. NO

17. (2 points) $(\forall x)Lax \wedge \neg Hb$

17. YES

0.4 Semantics

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, for all other names α $\mathscr{I}(\alpha) = 4$, $\mathscr{I}(M) = \{4\}$, $\mathscr{I}(H) = \{1, 3\}$ $\mathscr{I}(O) = \{1, 2, 3\}$

18. (2 points) Md

18. $v(Md) = T$

19. (2 points) $\neg Hd$

19. $v\neg(Hd) = T$

20. (2 points) $(\forall y)Oy$

20. $v(\forall y)Oy = F$

21. (2 points) $(\exists x)Mx$

21. $v(\exists x)Mx = T$

22. (2 points) $(\exists x)(Mx \wedge Ox)$

22. $v(\exists x)(Hx \wedge Ox) = F$

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4, 5, 6\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 3$, $\mathscr{I}(c) = 4$, $\mathscr{I}(d) = 5$, for all other names α $\mathscr{I}(\alpha) = 5$, $\mathscr{I}(Lxy) = \{\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 3, 2\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle\}$, $\mathscr{I}(Ax) = \{5\}$

23. (2 points) Laa

23. $v(Laa) = F$

24. (2 points) Lba

24. $v(Lba) = F$

25. (2 points) $(\forall x)Lax$

25. $v(\forall)Lax = F$

26. (2 points) $(\exists x)Lxx$

26. $v(\exists)Lxx = F$

27. (2 points) $(\forall x)Lxx \lor Ad$

27. $v(\forall x)Lxx \lor Ad = T$

0.5 Translation

Directions: Translate the following English sentences into the language of predicate logic. Write the formula on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a) =$ Al, $\mathscr{I}(f) =$ Frannie, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

28. (2 points) Al is not happy.

28. $\neg Ha$

29. (2 points) Al does not love Franny.

29. $\neg Laf$

30. (2 points) Someone is happy or someone is rich.

30. $(\exists x)Hx \lor (\exists x)Rx$

31. (2 points) Someone is both rich and happy.

31. $(\exists x)(Rx \land Hx)$

32. (2 points) All happy people are rich.

32. $(\forall x)(Hx \to Rx$

33. (2 points) If everyone loves Frannie, then Frannie loves someone.

33. $(\forall x)(Lxf) \to (\exists x)Lfx$

Directions: Translate the following predicate logic wffs into English. Write your translation on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a) =$ Al, $\mathscr{I}(f) =$ Frannie, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

34. (2 points) $(\exists x)(\neg Rx \land Hx)$

Solution: Some non-rich people are happy.

35. (2 points) $(\forall x)Lxa$

Solution: Everyone loves Al

36. (2 points) $(\forall x)(Rx \to \neg Hx)$

Solution: No rich people are happy.

37. (2 points) $(\forall x)(Lxx \to Hx)$

Solution: All people who love themselves are happy.

38. (2 points) $(\exists x)(Rx \wedge Lxx)$

Solution: Some rich people loves themselves.

0.6 Truth Trees and Models

Directions: Use a truth-tree to determine whether the following sets of wffs are consistent/inconsistent or arguments are valid/invalid. If the tree shows the set to be consistent or the argument to be invalid, construct a model illustrating this fact. (Rubric: Tree=5pts, Property=2pt, Model=5pts, if applicable)

39. (12 points) Determine consistent/inconsistent: $\neg(\forall y)Qy, Qb \wedge \neg Qc, (\exists x)(Px \wedge Qx)$

Solution: Hint: Setup the tree by writing $\neg(\forall y)Qy, Qb \wedge \neg Qc, (\exists x)(Px \wedge Qx)$ and then decompose the tree. The tree will be open and so the set of wffs is consistent. Since it is consistent, you will need to construct a model.

40. (12 points) $(\exists x)(Qx \wedge Px)$ therefore $(\exists x)(Qx \wedge Rx)$

Solution: Hint: Setup the tree by writing $(\exists x)(Qx \wedge Px)$ and then the negation of the conclusion: $\neg(\exists x)(Qx \wedge Rx)$. Then decompose the tree. The tree will be open, the argument is invalid. You will need to construct a model. Example Model: $D : \{1\}, \mathscr{I}(a) = 1 \mathscr{I}(P) = \{1\}, \mathscr{I}(Q) = \{1, \}, \mathscr{I}(R) = \{\}$

0.7 Bonus Questions

41. (1 point (bonus)) Translate the following argument into **RL**, use a tree to determine if it is valid or invalid (show your work). If it is invalid, construct a model (either formal or informal) demonstrating its invalidity. "If Frank murdered someone and Frank was drinking, then Frank will be arrested. Someone will arrest Frank. Therefore, Frank murdered someone."

42. (1 point (bonus)) What, if anything, is the *semantic* difference between $(\exists x)(\forall y)Lxy$ and $(\forall x)(\exists y)Lxy$?

$P \wedge Q$ P Q	 $\wedge D$ $\wedge D$	$P \vee Q$	P	Q	$\vee D$
$\neg(P \vee Q)$ $\neg(P)$ $\neg(Q)$	 $\neg \vee D$ $\neg \vee D$	$\neg(P \wedge Q)$	$\neg(P)$	$\neg(Q)$	$\neg \wedge D$
$\neg(P \rightarrow Q)$ P $\neg(Q)$	 $\neg \rightarrow D$ $\neg \rightarrow D$	$P \leftrightarrow Q$	P Q	$\neg(P)$ $\neg(Q)$	$\leftrightarrow D$ $\leftrightarrow D$
$\neg\neg(P)$ P	 $\neg\neg D$	$\neg(P \leftrightarrow Q)$	P $\neg(Q)$	$\neg(P)$ Q	$\neg \leftrightarrow D$ $\neg \leftrightarrow D$
$\neg(\exists x)\mathbf{P}\checkmark$ $(\forall x)\neg(\mathbf{P}), \neg\exists D$		$\neg(\forall x)\mathbf{P}\checkmark$ $(\exists x)\neg(\mathbf{P}), \neg\forall D$			
$(\exists x)\mathbf{P}\checkmark$ $\mathbf{P}(a/x), \exists D$		$(\forall x)\mathbf{P}$ $\mathbf{P}(a/x), \forall D$			

Table 1: Truth tree decomposition rules for **PL** and **RL**

Directions: Please write your **name** on the top of this page. Answer all of the questions on the answer sheet provided. If an answer will not fit on the blank provided, place your answer on one of the several blank pages.

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