Directions: This exam has 39 questions, for a total of 100 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

1 Multiple Choice

1. (2 points) An interpretation of **RL** is a function that does what (indicate all that apply):
   - A. specifies what objects are in the domain.
   - B. assigns truth values to n-place predicate terms followed by n terms.
   - C. FOR EACH NAME IN **RL** IT ASSIGNS THAT NAME ONE AND ONLY ONE ITEM IN $\mathcal{D}$
   - D. FOR EACH n-PLACE PREDICATE TERM IN **RL**ASSIGNS, IT ASSIGNS THAT PREDICATE TERM A SET OF n-TUPLES COMPOSED OF ELEMENTS FROM $\mathcal{D}$
   - E. assigns truth values to objects and wffs

2. (2 points) What is the principal weakness of **PL** in comparison to **RL**
   - A. **PL** IS NOT EXPRESSIVE ENOUGH: THERE ARE VALID ENGLISH ARGUMENTS THAT CAN BE EXPRESSED IN **RL** THAT CANNOT BE EXPRESSED IN **PL**
   - B. **PL** is too expressive: there are valid arguments in **PL** for which it would be impossible to express in English.
   - C. **PL** has an imprecise syntax, while the syntax of **RL** is fully precise.
   - D. **PL** has an imprecise semantics, while the semantics of **RL** is fully precise.

3. (2 points) What is a model ($\mathcal{D}$)?
   - A. A MODEL ($\mathcal{M}$) IS A TWO-PART STRUCTURE CONSISTING OF A DOMAIN ($\mathcal{D}$) AND AN INTERPRETATION FUNCTION ($\mathscr{I}$)
   - B. a model ($\mathcal{M}$) is a three-part structure consisting of a domain ($\mathcal{D}$), an interpretation function ($\mathscr{I}$), and a valuation (v) function.
   - C. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and a valuation function v where the valuation function assigns truth values to RL-wffs.
   - D. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$)

4. (2 points) In a predicate logic tree, under what conditions is a branch that contains a universally quantified wff (e.g. $(\forall x)Px$) considered a *completed open branch* (indicate all that apply)
   - A. when $(\forall x)Px$ has been decomposed into $\neg(\exists x)Px$
   - B. when $\Diamond P$ has been decomposed and relativized to a possible world, e.g., irj
   - C. WHEN $(\forall x)Px$ HAS BEEN DECOMPOSED FOR EVERY NAME $a, b, c, \ldots$ THAT OCCURS IN THAT BRANCH
   - D. WHEN ALL THE COMPLEX WFFS (NON-LITERALS) THAT ARE IN THAT BRANCH AND THAT CAN BE DECOMPOSED HAVE BEEN DECOMPOSED
   - E. WHEN THE BRANCH IS NOT CLOSED, VIZ., DOES NOT CONTAIN A WFF AND ITS LITERAL NEGATION

5. (2 points) What is a bound variable?
   - A. A variable is bound if and only if it is in the scope of a quantifier that quantifies for some variable.
   - B. A variable is bound if and only if it is a name.

C. A variable is bound if and only if it is in the scope of a quantifier.

D. A VARIABLE IS BOUND IF AND ONLY IF IT IS IN THE SCOPE OF A QUANTIFIER THAT QUANTIFIES FOR THAT VARIABLE.

6. (2 points) What does $\phi \models \psi$ mean?

A. $\phi \models \psi$ MEANS THAT THERE IS NO MODEL SUCH THAT ϕ IS TRUE AND ψ IS FALSE

B. $\phi \models \psi$ means that there is a proof of ψ from ϕ

C. $\phi \models \psi$ means that ϕ and ψ are RL-consistent

D. $\phi \models \psi$ means that ψ logically entails ϕ

2 Symbols

7. (2 points) Which of the following symbols are **RL** names (indicate all that apply)?

A. d

B. y

C. n

D. $\exists$

E. P

F. $\forall$

3 Syntax

Identify the main operator of the following wffs.

8. (2 points) $(\forall x)Lxx$

8. $\forall$

9. (2 points) $\neg(\exists x)Lxx$

9. $\neg$

10. (2 points) $(\exists y)(\forall x)(Lxy \wedge \neg Hx)$

10. $\exists$

State whether the following formulas are wffs. You can assume that Q is a one-place predicate, that L is a two-place predicate, and conventions for simplifying wffs are present.

11. (2 points) La

11. NO.

12. (2 points) Qx

12. WFF.

13. (2 points) Lab

13. Wff.

14. (2 points) $\neg Qa$

14. Wff.

15. (2 points) $(\forall x)Lxx$

15. Wff.

16. (2 points) $(\exists y)(\forall x)(Lxy \wedge \neg Lx)$

16. No.

4 Semantics

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, $\mathscr{I}(e) = 5$, for all other names α, $\mathscr{I}(\alpha) = 4$, $\mathscr{I}(N) = \{1, 2, 3, 4, 5\}$, $\mathscr{I}(G) = \{\langle 2, 1\rangle, \langle 3, 2\rangle, \langle 3, 1\rangle, \langle 5, 1\rangle\}$, $\mathscr{I}(I) = \{\}$, $\mathscr{I}(E) = \{2, 4\}$, $\mathscr{I}(O) = \{1, 3, 5\}$

17. (2 points) Ob

17. F

18. (2 points) $(\exists x)Ix$

18. F

19. (2 points) $(\forall x)Ex$

19. F

20. (2 points) $(\exists x)\neg Ex$

20. T

21. (2 points) $\neg(\exists x)Ex$

21. F

22. (2 points) $(\forall x)(Ex \rightarrow Nx)$

22. T

23. (2 points) $(\forall x)(Nx \rightarrow \neg Ex)$

23. F

24. (2 points) Gbd

24. F

25. (2 points) $(\exists x)Gex$

25. T

5 Translation

Directions: Translate the following English sentences into the language of predicate logic. Write the formula on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a)$ = Ava, $\mathscr{I}(j)$ = Jon, $\mathscr{I}(e)$ = Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

26. (2 points) Eve is not rich.

26. $\neg Re$

27. (2 points) Jon loves Eve.

27. Lje

28. (2 points) Someone is *both* rich and happy.

28. $(\exists x)(Rx \wedge Hx)$

29. (2 points) Someone is rich and someone is not happy.

29. $(\exists x)Rx \wedge (\exists x)\neg Hx$

30. (2 points) All happy people are rich.

30. $(\forall x)(Hx \rightarrow Rx)$

Directions: Translate the following predicate logic wffs into English. Write your translation on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a)$ = Ava, $\mathscr{I}(j)$ = Jon, $\mathscr{I}(e)$ = Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

31. (2 points) Ra

Solution: Ava is rich.

32. (2 points) $(\exists x)\neg Lxx$

Solution: Someone does not love themselves.

33. (2 points) $(\forall x)Lxa$

Solution: Everyone loves Ava.

34. (2 points) $(\exists x)(Hx \wedge \neg Rx)$

Solution: Someone is both happy and not rich.

35. (2 points) $(\forall x)(Hx \rightarrow Lxx)$

Solution: All happy people love themselves.

Directions: For each sentence, complete the following two tasks. First, answer what type of quantificational ambiguity is present in the sentence? Second, given that the sentence is quantificationally ambiguous, translate each of the two different interpretations of this sentence into predicate logic. Key: $Cx = x$ is a criminal, $Gx = x$ is greedy, $Bx = x$ is bad.

36. (5 points) Criminals are bad.

Solution: See handout and lecture notes. We did this one in class.

37. (5 points) All criminals are not greedy.

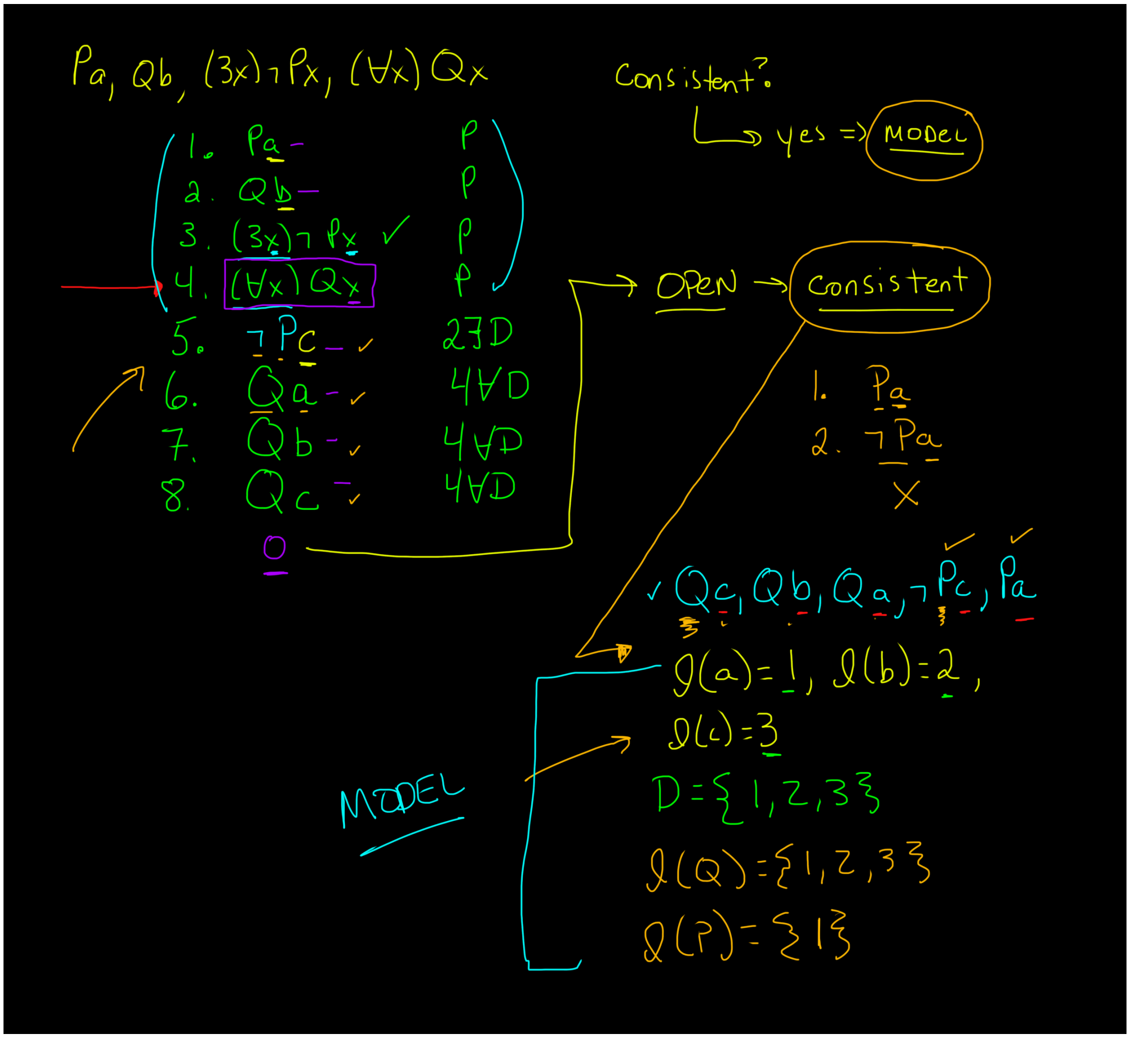
Solution: See handout and lecture notes. We did this one in class.

6 Trees

Directions: Use a truth-tree to determine whether the following sets of wffs are consistent/inconsistent or arguments are valid/invalid. If the tree shows the set to be consistent or the argument to be invalid, construct a model illustrating this fact. (Rubric: Tree=5pts, Property=1pt, Model=4pts, if applicable)

38. (10 points) Determine consistent/inconsistent: $Pa, Qb, (\exists x)\neg Px, (\forall x)Qx$

39. (10 points) Determine semantic entailment: $(\exists x)Px, (\exists x)Mx \vdash (\forall x)(Px \to Mx)$


Pa, Qb, (∃x)¬Px, (∀x)Qx
Consistent?
yes ⇒ MODEL
1. Pa P
2. Qb P
3. (∃x)¬Px P
4. (∀x)Qx P
5. ¬Pc 3∃D
6. Qa 4∀D
7. Qb 4∀D
8. Qc 4∀D
O
OPEN → Consistent
1. Pa
2. ¬Pa
X
Qc, Qb, Qa, ¬Pc, Pa
I(a) = 1, I(b) = 2,
I(c) = 3
D = {1, 2, 3}
I(Q) = {1, 2, 3}
I(P) = {1}
MODEL

$P \wedge Q$ P $\qquad \wedge D$ Q $\qquad \wedge D$	$P \vee Q$ $\swarrow \quad \searrow$ $P \qquad Q$ $\qquad \vee D$
$\neg(P \vee Q)$ $\neg(P)$ $\qquad \neg \vee D$ $\neg(Q)$ $\qquad \neg \vee D$	$\neg(P \wedge Q)$ $\swarrow \quad \searrow$ $\neg(P) \qquad \neg(Q)$ $\qquad \neg \wedge D$
$\neg(P \rightarrow Q)$ P $\qquad \neg \rightarrow D$ $\neg(Q)$ $\qquad \neg \rightarrow D$	$(P \rightarrow Q)$ $\swarrow \quad \searrow$ $\neg(P) \qquad Q$ $\qquad \rightarrow D$
$P \leftrightarrow Q$ $\swarrow \quad \searrow$ $P \qquad \neg(P)$ $\qquad \leftrightarrow D$ $Q \qquad \neg(Q)$ $\qquad \leftrightarrow D$	$\neg\neg(P)$ P $\qquad \neg\neg D$
$\neg(P \leftrightarrow Q)$ $\swarrow \quad \searrow$ $P \qquad \neg(P)$ $\qquad \neg \leftrightarrow D$ $\neg(Q) \qquad Q$ $\qquad \neg \leftrightarrow D$	
$\neg(\exists x)\phi$✓ $(\forall x)\neg(\phi), \neg\exists D$	$\neg(\forall x)\phi$✓ $(\exists x)\neg(\phi), \neg\forall D$
$(\exists x)\phi$✓ $\phi(\alpha/x), \exists D$	$(\forall x)\phi$ $\phi(\alpha/x), \forall D$

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