Directions: This exam has 28 questions, for a total of 100 points and 10 bonus points. Write your **name**, the **exam version**, and your **answers** on the answer sheet provided. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may write on this exam and may use the last pages of this exam as scrap paper.

Multiple Choice

Directions: Answer the questions in the spaces provided by circling one and only one answer (unless the question states otherwise).

1. (2 points) What is a deductive apparatus for **PL**?
   A. a set of rules that state that the rows in a proof need to be numbered.
   B. a set of rules that state how the proof is supposed to look, e.g. horizontally rather than vertically.
   C. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.
   D. It is a set of rules of reason that all people use to reason from one proposition to another, including, but not limited to, hypothetical and probabilistic reasoning.
   E. A SET OF DERIVATION RULES THAT EXPRESSES WHICH WFFS **Q** CAN BE WRITTEN AFTER WHICH WFFS **P** IN A DERIVATION.

2. (2 points) What is a derivation of **Q** from **Γ** using **PD**?
   A. A derivation of **Q** is a finite string of wffs starting with some premises **A**, **B**, **C**, . . . and ending with **Q**.
   B. A derivation of **Q** is a finite string of wffs starting with some premises **A**, **B**, **C**, . . . or assumptions and ending with **Q**.
   C. A DERIVATION OF **Q** IS A *finite* STRING OF FORMULAS FROM A SET Γ OF **PL** WFFS WHERE (I) THE LAST FORMULA IN THE STRING IS **Q** AND (II) EACH WFF IN THE SET IS EITHER A PREMISE, AN ASSUMPTION, OR IS THE RESULT OF THE PRECEDING WFFS AND THE DEDUCTIVE APPARATUS.
   D. A derivation of **Q** is an *infinite* string of formulas from a set Γ of **PL** wffs where (i) the last formula in the string is **Q** and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.
   E. A derivation of **Q** is an infinite string of wffs starting with some premises **A**, **B**, **C**, . . . or assumptions and ending with **Q**.

3. (2 points) In logic, there are two notions of logical consequence (entailment). The first notion is the semantic (model-theoretic) notion, which says that Q is a logical consequence of Γ if and only if there is no interpretation of the members of Γ and Q such that all of the members of Γ are true and Q is false. What is the other notion of logical consequence?
   A. THE SYNTACTIC (PROOF-THEORETIC) NOTION OF LOGICAL CONSEQUENCE (ENTAILMENT). THIS NOTION SAYS THAT Q IS A LOGICAL CONSEQUENCE OF Γ IF AND ONLY IF THERE IS A DERIVATION OF Q FROM Γ
   B. the semi-semantic notion of logical consequence (entailment). This notion says that Q is a semi-semantic consequence of Γ if and only if it is impossible for the premises to be true and the conclusion false, defined in terms of truth tables and trees.
   C. the intuitive notion of logical consequence (entailment). This notion says that Q is a *logical* consequence of Γ if and only if Q intuitively follows from Γ

D. the legal notion of logical consequence (entailment). This notion says that Q is a logical consequence of Γ if and only if Q would be accepted in a court of law or some practical matter.

E. none of the above

4. (2 points) What is the difference between $\Gamma \vdash \mathbf{Q}$ and $\Gamma \models \mathbf{Q}$?

A. $\Gamma \models \mathbf{Q}$ is syntactic consequence while $\Gamma \vdash \mathbf{Q}$ is semantic consequence.

B. $\Gamma \models \mathbf{Q}$ is hypostatic entailment while $\Gamma \vdash \mathbf{Q}$ is phenomenological entailment

C. $\Gamma \models \mathbf{Q}$ is phenomenological entailment while $\Gamma \vdash \mathbf{Q}$ is hypostatic entailment

D. $\Gamma \models \mathbf{Q}$ is syntactic consequence while $\Gamma \vdash \mathbf{Q}$ is semantic consequence.

E. $\Gamma \vdash \mathbf{Q}$ IS SYNTACTIC CONSEQUENCE WHILE $\Gamma \models \mathbf{Q}$ IS SEMANTIC CONSEQUENCE.

5. (2 points) What single derivation rule would allow you to reason to $X \wedge Z$ from $P, P \rightarrow (X \wedge Z)$?

A. $\rightarrow I$

B. $\leftrightarrow E$

C. $\wedge E$

D. MT

E. $\rightarrow E$

6. (2 points) What single derivation rule would allow you to reason to $\neg P \wedge \neg R$ from $\neg(P \vee R)$?

A. HS

B. MT

C. IMP

D. DN

E. DeM

7. (2 points) What single derivation rule would allow you to reason to $\neg(P \wedge R)$ from $\neg P \vee \neg R$?

A. HS

B. MT

C. IMP

D. DN

E. DeM

8. (2 points) What single derivation rule would allow you to reason to Z from $(A \wedge B) \leftrightarrow Z, A \wedge B$?

A. $\vee E$

B. $\rightarrow E$

C. $\wedge E$

D. $\leftrightarrow I$

E. $\leftrightarrow E$

9. (2 points) What single derivation rule would allow you to reason to $\neg P \vee Q$ from $P \rightarrow Q$?

A. HS

B. MT

C. DeM

D. DN

E. IMP

10. (2 points) What single derivation rule would allow you to reason to $A \to C$ from $A \to B$ and $B \to C$?
    A. HS
    B. MT
    C. DeM
    D. DN
    E. IMP

Short Answer

Directions: Answer the questions on the line provided by writing the abbreviation for the derivation rule (e.g. $\leftrightarrow E$ that is best described in the question prompt provided.

11. (2 points) What derivation rule is best described as follows: if on the assumption A, B and $\neg(B)$ is derived within the subproof, then $\neg(A)$ can be derived.

11. $\neg I$

12. (2 points) What single derivation rule would allow you to reason to $\neg C$ from $L \wedge \neg C$?

12. $\wedge E$

13. (2 points) What single derivation rule would allow you to reason to $M \vee (B \wedge Q)$ from M?

13. $\vee I$

14. (2 points) What single derivation rule would allow you to reason to M from $M \vee Q$ and $\neg Q$

14. DS

15. (2 points) What single derivation rule would allow you to reason to M from $M \leftrightarrow Q$ and Q

15. $\leftrightarrow E$

16. (2 points) What derivation rule is best described as follows: if $(P \to Q) \wedge Q$ is on a line of the proof, then it is legitimate to derive $P \to Q$ on a line and it is legitimate to derive Q on another line.

16. $\wedge E$

17. (2 points) What derivation rule best describes the following reasoning: If John watches Netflix, then Mary will go to the party. John watches Netflix. Therefore, Mary will go to the party.

17. $\to E$

18. (2 points) What derivation rule is best described as follows: if on the assumption $\neg(W)$ both $\neg A$ and A follow, then W can be derived.

18. $\neg E$

19. (2 points) What derivation rule is best described as follows: given $P \vee Q$, if P is assumed and it is shown that R follows from P and if Q is assumed and it is shown that R follows from Q, then R can be derived.

19. $\vee E$

20. (2 points) What derivation rule is best described as follows: given $(P \wedge Q) \vee M$ and $\neg(P \wedge Q)$, then M can be derived.

20. ______ DS ______

Derivations

Directions: Solve the following proofs. Be sure to setup the proof correctly, number all lines, and clearly indicate how each line is justified using the rules from the deductive apparatus.

21. (10 points) $A \wedge (Q \wedge T), T \rightarrow W, (A \wedge W) \rightarrow M \vdash M$

Solution: Use $\wedge E$ to derive T and A, then $\rightarrow E$ to derive W. With A and W, use $\wedge I$, then use $\rightarrow E$ to derive M.

22. (10 points) $(P \vee M) \rightarrow S, M \vdash S \wedge (M \vee Q)$

Solution:

23. (10 points) $A \rightarrow W, \neg W, A \vee M \vdash M$

Solution:

24. (10 points) $\neg(P \vee Q), \neg Q \rightarrow M \vdash M \vee \neg S$

Solution:

25. (10 points) $\vdash P \vee \neg P$

Solution:

26. (10 points) $\vdash P \rightarrow ((Q \wedge S) \rightarrow (S \wedge P))$

Solution:

Bonus Questions

27. (5 points (bonus)) $P \leftrightarrow S \dashv\vdash (P \wedge S) \vee (\neg P \wedge \neg S)$, 2 proofs

28. (5 points (bonus)) A set of derivation rules (e.g. $\rightarrow E, \wedge E$, etc.) is said to be **sound** if and only if (iff) for every syntactic entailment ($\Gamma \vdash Q$) there is also a semantic entailment ($\Gamma \models Q$). In short, if $\Gamma \vdash Q$, then $\Gamma \models Q$. Give a sketch of how you might prove that this is true.

PL Derivation Rules

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$
$P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction ($\rightarrow I$)

n	| P	A
$\vdots$	| $\vdots$	
$(n+1)$	| Q	
$(n+2)$	$P \rightarrow Q$	$\rightarrow I$, n–$(n+1)$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

n	| P	A
$\vdots$	| $\vdots$	
$(n+1)$	| Q	
$(n+2)$	| $\neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Derivation Rule – Negation Elimination ($\neg E$)

n	| $\neg(P)$	A
$\vdots$	| $\vdots$	
$(n+1)$	| Q	
$(n+2)$	| $\neg Q$	
$(n+3)$	P	$\neg E$, n–$(n+2)$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

1	$P \vee Q$	P
n	$\mid$ P	A
$\vdots$	$\mid$ $\vdots$	
$(n+1)$	$\mid$ R	
(i)	$\mid$ Q	A
$\vdots$	$\mid$ $\vdots$	
$(i+1)$	$\mid$ R	
(k)	R	$\vee E$, 1, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

n	$\mid$ P	A
$\vdots$	$\mid$ $\vdots$	
$(n+1)$	$\mid$ Q	
(i)	$\mid$ Q	A
$\vdots$	$\mid$ $\vdots$	
$(i+1)$	$\mid$ P	
(k)	$P \leftrightarrow Q$	$\leftrightarrow I$, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \vee Q, \neg Q \vdash P$ or $P \vee Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \rightarrow Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$

Derivation Rule – Implication (IMP)

$P \to Q \dashv\vdash \neg P \lor Q$

Directions: Please write your **name** on the top of the page. Please write clearly. **J**

1. ________________
2. ________________
3. ________________
4. ________________
5. ________________
6. ________________
7. ________________
8. ________________
9. ________________
10. ________________
11. ________________
12. ________________
13. ________________
14. ________________
15. ________________
16. ________________
17. ________________
18. ________________
19. ________________
20. ________________