Name: ______________________________

Directions: This exam has 29 questions, for a total of 100 points and 15 bonus points. Write your **name**, **exam version**, and your **answers** on the answer sheet provided. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may write on this exam and may use the last pages of this exam as scrap paper.

Short Answer

Directions: Answer the questions on the line provided by writing the abbreviation for the derivation rule (e.g. $\leftrightarrow E$ that is best described in the question prompt provided.

1. (2 points) What derivation rule is best described as follows: if on the assumption A, B and $\neg(B)$ is derived within the subproof, then $\neg(A)$ can be derived.

   1. ______________

2. (2 points) What derivation rule is best described as follows: if $(P \rightarrow Q) \wedge Q$ is on a line of the proof, then it is legitimate to derive derive $P \rightarrow Q$ on a line and it is legitimate to derive Q on another line.

   2. ______________

3. (2 points) What derivation rule best describes the following reasoning: If John will go to the party, then Mary will go to the party. John will go the party. Therefore, Mary will go to the party.

   3. ______________

4. (2 points) What derivation rule best describes the following reasoning: If John invests in risky stocks, then he will either get rich or he will go broke. John neither got rich nor did he go broke. Therefore, John did not invest in risky stocks.

   4. ______________

5. (2 points) What derivation rule is best described as follows: if on the assumption $\neg(W)$ both $\neg A$ and A follow, then W can be derived.

   5. ______________

6. (2 points) What derivation rule is best described as follows: given $P \vee Q$, if P is assumed and it is shown that R follows from P and if Q is assumed and it is shown that R follows from Q, then R can be derived.

   6. ______________

7. (2 points) What derivation rule is best described as follows: given $(P \wedge Q) \vee M$ and $\neg(P \wedge Q)$, then M can be derived.

   7. ______________

Multiple Choice

Directions: Answer the questions in the spaces provided by circling one and only one answer (unless the question states otherwise).

8. (2 points) What is a deductive apparatus for **PL**?
    A. a set of rules that state that the rows in a proof need to be numbered.
    B. a set of rules that state how the proof is supposed to look, e.g. horizontally rather than vertically.
    C. a set of derivation rules that expresses which wffs **Q** can be written after which wffs **P** in a derivation.
    D. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.

9. (2 points) What is a derivation of **Q** using **PD**?
    A. A derivation of **Q** is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with **Q**.
    B. A derivation of **Q** is a finite string of formulas from a set Γ of **PL** wffs where (i) the last formula in the string is **Q** and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.
    C. A derivation of **Q** is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.
    D. A derivation of **Q** is an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.

10. (2 points) In logic, there are two notions of logical consequence (entailment). The first notion is the semantic (model-theoretic) notion, which says that Q is a logical consequence of Γ if and only if there is no interpretation of the members of Γ and Q such that all of the members of Γ are true and Q is false. What is the other notion of logical consequence?
    A. the syntactic (proof-theoretic) notion of logical consequence (entailment). This notion says that Q is a logical consequence of Γ if and only if there is a derivation of Q from Γ
    B. the semi-semantic notion of logical consequence (entailment). This notion says that Q is a semi-semantic consequence of Γ if and only if it is impossible for the premises to be true and the conclusion false, defined in terms of truth tables and trees.
    C. the intuitive notion of logical consequence (entailment). This notion says that Q is a *logical* consequence of Γ if and only if Q intuitively follows from Γ
    D. the legal notion of logical consequence (entailment). This notion says that Q is a logical consequence of Γ if and only if Q would be accepted in a court of law or some practical matter.
    E. none of the above

11. (2 points) What is the difference between $\Gamma \vdash \mathbf{Q}$ and $\Gamma \models \mathbf{Q}$?
    A. $\Gamma \models \mathbf{Q}$ is syntactic consequence while $\Gamma \vdash \mathbf{Q}$ is semantic consequence.
    B. $\Gamma \vdash \mathbf{Q}$ is syntactic consequence while $\Gamma \models \mathbf{Q}$ is semantic consequence.

12. (2 points) What single derivation rule would allow you to reason to $X \wedge Z$ from $P, P \rightarrow (X \wedge Z)$?
    A. $\rightarrow I$
    B. $\leftrightarrow E$
    C. $\wedge E$
    D. MT
    E. $\rightarrow E$

13. (2 points) What single derivation rule would allow you to reason to $\neg P \wedge \neg R$ from $\neg(P \vee R)$?
    A. HS

B. MT

C. IMP

D. DN

E. DeM

14. (2 points) What single derivation rule would allow you to reason to P from $Q \rightarrow P, Q$?

A. HS

B. MT

C. IMP

D. DN

E. $\rightarrow E$

15. (2 points) What single derivation rule would allow you to reason to $\neg(Q \vee P)$ from $(Q \vee P) \rightarrow S$ and $\neg S$?

A. HS

B. MT

C. IMP

D. DN

E. DeM

16. (2 points) What single derivation rule would allow you to reason to Z from $(A \wedge B) \leftrightarrow Z$ and $A \wedge B$?

A. $\vee E$

B. $\rightarrow E$

C. $\wedge E$

D. $\leftrightarrow I$

E. $\leftrightarrow E$

17. (2 points) What single derivation rule would allow you to reason to $\neg P \vee Q$ from $P \rightarrow Q$?

A. HS

B. MT

C. DeM

D. DN

E. IMP

18. (2 points) What single derivation rule would allow you to reason to $A \rightarrow C$ from $A \rightarrow B$ and $B \rightarrow C$?

A. HS

B. MT

C. DeM

D. DN

E. IMP

19. (2 points) What single derivation rule would allow you to reason to $\neg C$ from $L \wedge \neg C$?

A. $\wedge E$

B. $\vee E$

C. $\rightarrow E$

D. $\leftrightarrow E$

E. DN

20. (2 points) What single derivation rule would allow you to reason to $A \vee (B \wedge Q)$ from A?

A. $\vee I$

B. $\rightarrow E$

C. $\leftrightarrow E$

D. $\wedge E$

E. DN

Derivations

Directions: Solve the following proofs. Be sure to setup the proof correctly, number all lines, and clearly indicate how each line is justified using the rules from the deductive apparatus.

21. (10 points) $A \wedge (B \wedge C), M \wedge Z \vdash B \wedge Z$

22. (10 points) $Z \rightarrow (Q \wedge T), Q \rightarrow M, Z \wedge L \vdash M$

23. (10 points) $\neg B, A \rightarrow B, (M \wedge C) \vee A \vdash C$

24. (10 points) $\neg(P \rightarrow Q) \vdash \neg Q$

25. (10 points) $\vdash \neg(P \wedge \neg P)$

26. (10 points) $\vdash (X \wedge T) \rightarrow (Q \rightarrow (X \vee \neg\neg T))$

Extra Credit

27. (5 points (bonus)) Devise a derivation rule for negated biconditionals (that is, wffs of the form: $\neg(\mathbf{P} \leftrightarrow \mathbf{Q})$. First, write down the derivation rule, e.g. $\mathbf{P} \wedge \mathbf{Q} \vdash \mathbf{Q}$. Second, name your derivation rule. Third, justify this derivation rule's inclusion by either (i) showing how it can be proved using the existing rules in the deductive apparatus or (ii) show how it is valid using a truth table.

28. (5 points (bonus)) A set of wffs Γ semantically entails a wff Q if and only if (iff) there is no interpretation such that the members of Γ are true and Q is false. That is, there is no interpretation such that $v(\gamma) = T$, where $\gamma \in \Gamma$ and $v(Q) = F$. This is expressed by writing $\Gamma \models Q$. A wff Q is said to be a tautology iff there is no interpretation of Q where Q is false. That is, $v(Q) = T$ under every interpretation. Can every $\Gamma \models Q$ be rewritten as a tautology? If so, what tautology?

29. (5 points (bonus)) A system of logic is said to be **sound** if and only if (iff) for every syntactic entailment ($\Gamma \vdash Q$) is also a semantic entailment ($\Gamma \models Q$). In short, if $\Gamma \vdash Q$, then $\Gamma \models Q$. Give a sketch of how you might prove that this is true.

Evaluation

Page:	1	2	3	4	Total
Points:	14	12	12	62	100
Bonus Points:	0	0	0	15	15
Score:					

PL Derivation Rules

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$
$P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction ($\rightarrow I$)

n	$\mid \underline{P}$	A
$\vdots$	$\mid \vdots$	
$(n+1)$	$\mid Q$	
$(n+2)$	$P \rightarrow Q$	$\rightarrow I$, n–$(n+1)$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

n	$\mid \underline{P}$	A
$\vdots$	$\mid \vdots$	
$(n+1)$	$\mid Q$	
$(n+2)$	$\mid \neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Derivation Rule – Negation Elimination ($\neg E$)

n	$\mid \underline{\neg(P)}$	A
$\vdots$	$\mid \vdots$	
$(n+1)$	$\mid Q$	
$(n+2)$	$\mid \neg Q$	
$(n+3)$	P	$\neg E$, n–$(n+2)$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

1	$P \vee Q$	P
n	$\quad$ | P	A
$\vdots$	$\quad$ | $\vdots$	
$(n+1)$	$\quad$ | R	
(i)	$\quad$ | Q	A
$\vdots$	$\quad$ | $\vdots$	
$(i+1)$	$\quad$ | R	
(k)	R	$\vee E$, 1, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

n	$\quad$ | P	A
$\vdots$	$\quad$ | $\vdots$	
$(n+1)$	$\quad$ | Q	
(i)	$\quad$ | Q	A
$\vdots$	$\quad$ | $\vdots$	
$(i+1)$	$\quad$ | P	
(k)	$P \leftrightarrow Q$	$\leftrightarrow I$, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \vee Q, \neg Q \vdash P$ or $P \vee Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \to Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \to Q, Q \to R \vdash P \to R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$

Derivation Rule – Implication (IMP)

$P \to Q \dashv\vdash \neg P \vee Q$