Name: _____

Directions: This exam has 22 questions, for a total of 99 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. YOu may use the last pages of this exam as scrap paper.

Short Answer

Directions: Answer the questions on the line provided by writing the abbreviation for the derivation rule that is best described in the question prompt provided.

1. (3 points) What derivation rule is best described as follows: if on the assumption P both $\neg Q$ and Q follow, then $\neg(P)$ can be derived.

2. (3 points) What derivation rule is best described as follows: if on the assumption P, Q is derived within the subproof, then $P \rightarrow Q$ can be derived.

3. (3 points) What derivation rule is best described as follows: given $P \vee Q$, if P is assumed and it is shown that R follows from P and if Q is assumed and it is shown that R follows from Q, then R can be derived.

4. (3 points) What derivation rule is best described as follows: on the assumption P, it follows that Q. And, on the assumption that Q, it follows that P. Therefore, $P \leftrightarrow Q$.

5. (3 points) What derivation rule best describes the following reasoning: : if John goes to the store, then he will buy some candy. John went to the store. Therefore, John bought some candy.

6. (3 points) What derivation rule best describes the following reasoning: if John goes to the store, then he will buy candy. John did not buy candy. Therefore, John did not go to the store.

7. (3 points) Can the following derivation be justified by any single derivation rule in the deductive apparatus for PL: "If John passes the exam, he will graduate. John did not pass the exam. Therefore, John will not graduate." Answer: "yes" (and list the derivation rule) or "no" (no need to list the rule).

Multiple Choice

Directions: Answer the questions in the spaces provided by circling one and only one answer (unless the question states otherwise).

8. (2 points) What is a deductive apparatus for **PL**?
   A. a set of derivation rules that expresses which wffs **Q** can be written after which wffs **P** in a proof.
   B. a set of rules that state that the rows in a proof need to be numbered.
   C. a set of rules that state how the proof is supposed to look, e.g. horizontally rather than vertically.
   D. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.

9. (2 points) What is a derivation of **Q** using **PD**?
   A. A derivation of **Q** is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with **Q**.

B. A derivation of **Q** is a finite string of formulas from a set Γ of **PL** wffs where (i) the last formula in the string is **Q** and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.

C. A derivation of **Q** is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.

D. A derivation of **Q** is an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.

10. (2 points) What is the difference between $\Gamma \vdash \mathbf{Q}$ and $\Gamma \models \mathbf{Q}$?

A. $\Gamma \models \mathbf{Q}$ is syntactic consequence while $\Gamma \vdash \mathbf{Q}$ is semantic consequence.

B. $\Gamma \vdash \mathbf{Q}$ is syntactic consequence while $\Gamma \models \mathbf{Q}$ is semantic consequence.

11. (2 points) What single derivation rule would allow you to reason to Q from $P \wedge Q$?

A. $\vee E$

B. $\wedge E$

C. $\rightarrow E$

D. $\leftrightarrow E$

12. (2 points) What single derivation rule would allow you to reason to $P \vee Q$ from Q?

A. $\vee I$

B. $\rightarrow E$

C. $\leftrightarrow E$

D. $\wedge E$

13. (2 points) What single derivation rule would allow you to reason to Q from $P, P \rightarrow Q$?

A. $\rightarrow I$

B. $\rightarrow E$

C. $\leftrightarrow E$

D. $\wedge E$

14. (2 points) What single derivation rule would allow you to reason to $\neg P$ from $P \rightarrow Q, \neg Q$?

A. HS

B. MT

C. DeM

D. IMP

15. (2 points) What single derivation rule would allow you to reason to $\neg P \wedge \neg R$ from $\neg(P \vee R)$?

A. HS

B. DeM

C. MT

D. IMP

16. (2 points) What single derivation rule would allow you to reason to $\neg P \vee Q$ from $P \rightarrow Q$?

A. HS

B. MT

C. DeM

D. IMP

Essay

Directions: Solve the following proofs.

17. (10 points) $A, B, C, ((A \wedge B) \wedge C) \rightarrow M \vdash M$
18. (10 points) $Z \rightarrow (M \rightarrow R), Z \wedge M \vdash R$
19. (10 points) $P \rightarrow M, \neg M, (Q \wedge T) \vee P \vdash Q$
20. (10 points) $\neg(Z \vee T), \neg Z \leftrightarrow \neg(Q \vee R) \vdash \neg Q$
21. (10 points) $\vdash (M \vee T) \rightarrow (P \vee \neg P)$
22. (10 points) $\vdash M \rightarrow (M \vee S)$

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$
$P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction ($\rightarrow I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$P \rightarrow Q$	$\rightarrow I$, n–$(n+1)$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$\neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Derivation Rule – Negation Elimination ($\neg E$)

n		$\neg(P)$	A
$\vdots$		$\vdots$	
$(n+1)$		Q	
$(n+2)$		$\neg Q$	
$(n+3)$	P		$\neg E$, n–$(n+2)$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

1	$P \vee Q$		P
n		P	A
$\vdots$		$\vdots$	
$(n+1)$		R	
(i)		Q	A
$\vdots$		$\vdots$	
$(i+1)$		R	
(k)	R		$\vee E$, 1, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

n		P	A
$\vdots$		$\vdots$	
$(n+1)$		Q	
(i)		Q	A
$\vdots$		$\vdots$	
$(i+1)$		P	
(k)	$P \leftrightarrow Q$		$\leftrightarrow I$, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \vee Q, \neg Q \vdash P$ or $P \vee Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \to Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \to Q, Q \to R \vdash P \to R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$

Derivation Rule – Implication (IMP)

$P \to Q \dashv\vdash \neg P \vee Q$

Evaluation

Page:	1	2	3	Total
Points:	25	14	60	99
Bonus Points:	0	0	0	0
Score:				