Directions: The following exam consists of 38 questions, for a total of 100 points and 8 bonus points. Read each question carefully (note: answers may break onto the next page).

This exam tests your knowledge over the material from Chapter 3 and Chapter 4 of *Symbolic Logic: Syntax, Semantics, and Proof* and any additional handouts or discussion generated from this text. While you may write on the test itself, please place your final answers on the "answer sheet" provided (you may use the back of the "answer sheet" as scrap paper).

For each question, choose one and only one (the best) answer (unless the question states otherwise).

1 Chapter 3: Truth Tables

1.1 Definitions, Concepts, and Basic Mechanics

1. (2 points) What is a decision procedure?
   - A. It is the actual decision a human being makes with respect to whether an argument has a particular logical property, e.g. judging an argument to be valid or invalid.
   - B. A mechanical method that determines (in a finite number of steps) whether a proposition, set of propositions, or argument has a certain logical property.
   - C. It is a step-by-step procedure used by logicians to translate a sentence from a natural language (e.g. English) into a formal language (e.g. propositional logic).
   - D. It is a psychological procedure whereby people made decisions about whether an argument is good or bad.
2. (2 points) What is a truth table?
   - A. A truth table is a table for determining the truth of a proposition, viz., whether it is actually true or actually false.
   - B. A truth table is a table that provides a graphical way of representing valuations of wffs(s) under a set of interpretations.
   - C. A truth table is a table for determining whether an argument is good or bad in terms of whether the argument is true or false.
   - D. A truth table is a table that is used to determine whether an argument is true or false.
3. (2 points) An argument is deductively valid in **PL** if and only if ...
   - A. It is impossible for both the premises and the negation of the conclusion to be true
   - B. it is possible for the premises to be true and the conclusion false
   - C. it is impossible for the premises and the conclusion to be false
   - D. it is possible for the conclusion to be true and the premises to be false.
   - E. it is impossible for the premises and the conclusion to be true.
4. (2 points) An argument $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ therefore $\mathbf{Z}$ is valid in **PL** if and only if ...
   - A. as a matter of fact, $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are true and Z is false.
   - B. as a matter of fact, $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are true and Z is true.
   - C. as a matter of fact, $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are false and Z is true.
   - D. as a matter of fact, $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are false and Z is true.
   - E. $\mathbf{Z}$ is a semantic consequence of $\mathbf{P}, \mathbf{Q}, \mathbf{R}$.

5. (2 points) An set of wffs $\{A, B, C, D\}$ semantically entails Q ($A, B, C, D \models Q$) if and only if what?
   - A. there is **no** interpretation such that each of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.
   - B. there is **an** interpretation such that each of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.
   - C. there is **at least two** interpretations such that each of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.
   - D. there is **no** interpretation such that each of the members (wffs) of $\{A, B, C, D\}$ are false and Q is true.
   - E. some of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.

1.2 Determining the truth of wffs

6. (2 points) Suppose $\mathscr{I}(P) = T$, what is $v(\neg\neg P)$?
7. (2 points) Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = F$, what is $v(P \to Q)$?
8. (2 points) Suppose $\mathscr{I}(P) = T$ and $\mathscr{I}(Q) = F$, what is $v(P \wedge Q)$?
9. (2 points) Suppose $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = T$, what is $v(P \leftrightarrow Q)$?
10. (2 points) $\mathscr{I}(P) = T$, $\mathscr{I}(R) = F$, and $\mathscr{I}(Q) = F$, determine the truth value of $(P \to \neg Q) \wedge R$
11. (2 points) Suppose $\mathscr{I}(P) = T, \mathscr{I}(Q) = F, \mathscr{I}(R) = F$, what is $v(P \wedge \neg Q) \wedge \neg R)$?
12. (2 points) What is the truth value of $P \vee \neg P$?
13. (2 points) What is the truth value of $\neg P \wedge P$?

1.3 Truth Tables: Construction

14. (10 points) On the answer sheet, construct a truth table for the following proposition and determine whether it is a contingency, tautology, or contradiction: $(\neg P \vee Q) \wedge (Q \to P)$. To receive full credit, you must (i) construct the entire truth table, (ii) label whether it is a contingency, tautology, or contradiction, and (iii) clearly explain why the table shows the wff in question has the property you say it does.
15. (10 points) On the answer sheet, construct a truth table for the following argument: $(\neg P \wedge Q) \to \neg Q, \neg Q \vee P$ therefore $P \to Q$. To receive full credit, you must (i) construct the entire truth table, (ii) label whether it is a valid or invalid, and (iii) clearly explain why the table shows the argument is valid or invalid.

2 Chapter 4: Truth Trees

2.1 Concepts, Setup, Terminology

16. (2 points) What is one advantage of truth trees over truth tables?
    - A. trees provide the user a more graphical way of seeing the truth or falsity of an argument, specifically by showing whether an argument is true or false under every interpretation
    - B. In contrast to truth tables where the complexity of the table is a function of the number of propositional letters (more letters, more rows required), the complexity of a truth tree is not a function of the number of propositional letters.

C. Tree trees are capable of showing more arguments to be valid (invalid) than truth tables.

D. In contrast to truth tables, truth trees are capable of representing (translating) more arguments into the language of propositional logic.

17. (2 points) Suppose the following literal wffs were found in a completed open branch, how would you assign an interpretation to the propositional letters that compose these wffs: $\neg P, Q, \neg R$

A. $\mathscr{I}(P) = T, \mathscr{I}(Q) = T, \mathscr{I}(R) = T$

B. $\mathscr{I}(P) = T, \mathscr{I}(Q) = T, \mathscr{I}(R) = F$

C. $\mathscr{I}(P) = F, \mathscr{I}(Q) = T, \mathscr{I}(R) = F$

D. $\mathscr{I}(P) = F, \mathscr{I}(Q) = F, \mathscr{I}(R) = F$

18. (2 points) What is a closed branch of a truth tree?

A. a closed branch is a branch of a tree is when all of the wffs in all of the branchs contain a wff $\mathbf{P}$ and its literal negation $\neg(\mathbf{P})$

B. a closed branch is when there is a way of assigning truth values to propositional letters in the branch that would make all of the wffs true, e.g. $\mathscr{I}(P) = T, \mathscr{I}(Q) = F$

C. a closed branch is a branch of a tree containing a wff $\mathbf{P}$ and its literal negation $\neg(\mathbf{P})$

D. a closed branch is a branch of a tree that does **not** contain a wff $\mathbf{P}$ and its literal negation $\neg(\mathbf{P})$

2.2 Construction and decomposition

19. (2 points) Suppose we were to test the following argument to determine whether or not it is deductively valid: $P \wedge R, R \rightarrow (S \vee \neg\neg T) \vdash \neg M$. The first step of using the truth-tree test is to setup the tree into what is called the “stack”. Which one of the following formulas would NOT be in the initial stack:

A. $P \wedge R$

B. $R \rightarrow (S \vee \neg\neg T)$

C. $\neg\neg M$

D. $\neg M$

20. (2 points) Suppose we were to test the following wff to determine whether it is a tautology: $P \rightarrow (Q \vee R)$. What wff would you use to test $P \rightarrow (Q \vee R)$? That is, what should the first line of your truth tree be?

A. $P \rightarrow (Q \vee R)$

B. $\neg P \rightarrow (Q \vee R)$

C. $\neg\neg P \rightarrow (Q \vee R)$

D. $\neg P \rightarrow (\neg Q \vee \neg R)$

E. $\neg(P \rightarrow (Q \vee R))$

21. (2 points) Suppose we were to test the following argument to determine whether or not it is deductively valid: $P \vee \neg R, \neg R \leftrightarrow (S \vee \neg T) \vdash \neg M \vee T$. The first step of using the truth-tree test is to setup the tree into what is called the “stack”. Which one of the following formulas would NOT be in the initial stack:

A. $P \vee \neg R$

B. $\neg R \leftrightarrow (S \vee \neg T)$

C. $\neg(\neg M \vee T)$

D. $\neg\neg M \vee T$

22. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg P \vee \neg Q$

23. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg(\neg R \to R)$

24. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg R \to R$

25. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg P \leftrightarrow \neg Q$?

26. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg R \vee \neg R$?

27. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg(P \wedge \neg Q)$?

28. (2 points) What is the first truth-tree decomposition rule that should be used on the following wff: $\neg(P \to \neg Q)$?

2.3 Truth Trees: Construction

29. (10 points) On the answer sheet, construct a truth tree for the following set of wffs and determine whether the set is consistent: $\neg P \wedge (\neg Q \vee P), Z \vee \neg(M \wedge Q), \neg Z \vee Q$. To receive full credit, you must (i) construct the entire truth tree, (ii) label whether it is a consistent or inconsistent, and (iii) either demonstrate that the tree is consistent by recovering an interpretation from the tree *or* clearly explaining why the tree shows the set to be inconsistent.

30. (10 points) On the answer sheet, construct a truth tree for the following set of wffs and determine whether $(P \vee Q) \to \neg(S \vee T), (S \wedge T) \wedge Q$ therefore $Z \vee \neg Q$ is valid. To receive full credit, you must (i) construct the entire truth tree, (ii) label whether it is valid or invalid, and (iii) either demonstrate that the tree is invalid by recovering an interpretation from the tree that shows the argument to be invalid *or* clearly explaining why the tree is valid.

3 Conceptual Questions

31. (2 points) What advantage does the truth table and truth tree tests have over the imagination test for validity?
    - A. The truth table/tree tests are mechanical (decision procedures)
    - B. the truth table/tree tests are poetic; they take into account the spirit of human nature
    - C. If an argument is deductively valid in English, then the truth table/tree method will always correctly determine whether it is (in fact) valid in the language of propositional logic (PL).
    - D. If an argument is persuasive in English, then the table/tree methods will tell us whether we ought to be persuaded by them.

32. (2 points) Suppose an argument has a tautology for a conclusion, what would the truth tree for the argument show? That is, would the tree be open or closed? Would the argument be valid or invalid?
    - A. The tree would have at least one open branch. The open branch would indicate that there is a way of interpreting the propositional letters so that it would be possible for the premises to be true and the conclusion false. Therefore, the argument in question would be *invalid*.
    - B. The tree would have at least one open branch. The open branch would indicate that there is a way of interpreting the propositional letters so that it would be possible for the premises to be true and the conclusion false. Therefore, the argument in question would be *valid*.

C. The tree would close. The closed tree would indicate that there is no way of interpreting the propositional letters that would make the premises true and the conclusion false. Therefore, the argument in question would be *valid*.

D. The tree would close. The closed tree would indicate that there is no way of interpreting the propositional letters that would make the premises true and the conclusion false. Therefore, the argument in question would be *invalid*.

E. Whether the tree would be open or closed would depend upon the premises. Given that we don't know what the premises of the argument are, we cannot say with any certainty whether the argument would be valid or invalid.

33. (2 points) Suppose that the premises of an argument are inconsistent, what would the truth tree for the argument show? That is, would the tree be open or closed? Would the argument be valid or invalid?

    A. Open and invalid.

    B. Open and valid.

    C. Closed and valid.

    D. Closed and invalid.

    E. Whether the tree would be open or closed would depend upon the premises. Given that we don't know what the premises of the argument are, we cannot say with any certainty whether the argument would be valid or invalid.

34. (2 points) If $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ is inconsistent, is $(\mathbf{P} \wedge \mathbf{Q}) \wedge \mathbf{R}$ a contradiction?

    A. No. If $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ is inconsistent, then it is possible for $(\mathbf{P} \wedge \mathbf{Q}) \wedge \mathbf{R}$ to be true.

    B. No. If $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ is inconsistent, then $(\mathbf{P} \wedge \mathbf{Q}) \wedge \mathbf{R}$ is a tautology.

    C. Yes.

4 Extra-Credit Questions

Directions: You do not need to answer the following questions. Answering them is purely for extra-credit.

35. (1 point (bonus)) If $\mathbf{P} \to \mathbf{Q}$ is a contradiction, is it the case that $\mathbf{P} \not\models \mathbf{Q}$? Explain your answer.

36. (3 points (bonus)) If $\mathbf{P} \models \mathbf{Q}$, then is it the case that $\mathbf{P} \to \mathbf{Q}$ is a tautology? Explain your answer.

37. (1 point (bonus)) If $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ is inconsistent, is $(\mathbf{P} \wedge \mathbf{Q}) \wedge \mathbf{R}$ a contradiction? Explain your reasoning.

38. (3 points (bonus)) If $\mathbf{P} \models \mathbf{Q}$, then is it always the case that $\mathbf{Q} \models \mathbf{P}$? Explain and illustrate your answer.

Congratulations!

- turn in your answer sheet at the front of the classroom,
- check to see if your homework has been graded,
- feel free to leave. Class is complete.
- answers to the exam will be posted to CANVAS.

$P \wedge Q$ P $\wedge D$ Q $\wedge D$	$P \vee Q$ $P \swarrow \quad \searrow Q$ $\vee D$
$\neg(P \vee Q)$ $\neg(P)$ $\neg \vee D$ $\neg(Q)$ $\neg \vee D$	$\neg(P \wedge Q)$ $\neg(P) \swarrow \quad \searrow \neg(Q)$ $\neg \wedge D$
$\neg(P \to Q)$ P $\neg \to D$ $\neg(Q)$ $\neg \to D$	$P \leftrightarrow Q$ $P \swarrow \quad \searrow \neg(P)$ $\leftrightarrow D$ $Q \quad\quad \neg(Q)$ $\leftrightarrow D$
$\neg\neg(P)$ P $\neg\neg D$	$\neg(P \leftrightarrow Q)$ $P \swarrow \quad \searrow \neg(P)$ $\neg \leftrightarrow D$ $\neg(Q) \quad\quad Q$ $\neg \leftrightarrow D$
	$(P \to Q)$ $\neg(P) \swarrow \quad \searrow Q$ $\to D$

Table 1: Truth tree decomposition rules for **PL**

P	$\neg P$
T	F
F	T

Table 2: Truth Table: Negation

P	R	$P \wedge R$	$P \vee R$	$P \to R$	$P \leftrightarrow R$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Table 3: Truth Table: Conjunction, Disjunction, Conditional, and Biconditional

Directions: Please write your **name** on the top of the page. Please write clearly. **I**

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