Directions: The following exam consists of 34 questions, for a total of 100 points and 0 bonus points. Read each question carefully (note: answers may break onto the next page). This exam tests your knowledge over the material from Chapter 3 and Chapter 4 of *Symbolic Logic: Syntax, Semantics, and Proof*, lectures, videos, handouts, and discussion. You may write on the test itself, but place final answers on the "answer sheet" provided. *For each question, choose one and only one (the best) answer (unless the question states otherwise).*

0.1 Definitions, Concepts, and Basic Mechanics

1. (2 points) What is a decision procedure?
    A. It is the actual decision a human being makes with respect to whether an argument has a particular logical property, e.g. judging an argument to be valid or invalid.
    B. It is a step-by-step procedure used by logicians to translate a sentence from a natural language (e.g. English) into a formal language (e.g. propositional logic).
    C. It is a psychological procedure whereby people made decisions about whether an argument is good or bad.
    D. A mechanical method that determines (in a finite number of steps) whether a proposition, set of propositions, or argument has a certain logical property.

2. (2 points) An set of wffs $\{A, B, C, D\}$ semantically entails Q if and only if what? That is, $A, B, C, D \models Q$ if and only if what?
    A. there is **no** interpretation such that each of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.
    B. there is **an** interpretation such that each of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.
    C. there is **at least two** interpretations such that each of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.
    D. there is **no** interpretation such that each of the members (wffs) of $\{A, B, C, D\}$ are false and Q is true.
    E. some of the members (wffs) of $\{A, B, C, D\}$ are true and Q is false.

3. (2 points) What advantage does the truth table and truth tree tests have over the imagination test for validity?
    A. The truth table/tree tests are mechanical (decision procedures)
    B. the truth table/tree tests are poetic; they take into account the spirit of human nature
    C. If an argument is deductively valid in English, then the truth table/tree method will always correctly determine whether it is (in fact) valid in the language of propositional logic (PL).
    D. If an argument is persuasive in English, then the table/tree methods will tell us whether we ought to be persuaded by them.

4. (2 points) What is one advantage of truth trees over truth tables?
    A. trees provide the user a more graphical way of seeing the truth or falsity of an argument, specifically by showing whether an argument is true or false under every interpretation
    B. In contrast to truth tables where the complexity of the table is a function of the number of propositional letters (more letters, more rows required), the complexity of a truth tree is not a function of the number of propositional letters.
    C. Tree trees are capable of showing more arguments to be valid (invalid) than truth tables.
    D. In contrast to truth tables, truth trees are capable of representing (translating) more arguments into the language of propositional logic.

5. (2 points) Suppose that a set of wffs is inconsistent, what would its truth tree show?
    A. Open, therefore consistent
    B. Open, therefore inconsistent
    C. Closed, therefore inconsistent
    D. Closed, therefore consistent
6. (2 points) How many truth tree tests are required to determine whether a wff is a contingency?

0.2 Construction and decomposition

7. (2 points) Suppose we were to test the following argument to determine whether or not it is deductively valid: $P \vee R, \neg P \rightarrow Q \vdash \neg M$. The first step of using the truth-tree test is to setup the tree into what is called the "stack". Which one of the following formulas would **NOT** be in the initial stack:
    A. $P \vee R$
    B. $\neg P \rightarrow Q$
    C. $\neg\neg M$
    D. $\neg M$
8. (2 points) Suppose we were to test the following argument to determine whether or not it is deductively valid: $P \rightarrow Q, \neg(P \vee Q) \models P \wedge S$. The first step of using the truth-tree test is to setup the tree into what is called the "stack". Which one of the following formulas would **NOT** be in the initial stack:
    A. $P \rightarrow Q$
    B. $\neg(P \vee Q)$
    C. $\neg(P \wedge S)$
    D. $P \wedge S$

0.3 Determining the truth of wffs

Directions: Determine the truth value (T or F):

9. (2 points) Suppose $\mathscr{I}(P) = T$, what is $v(\neg P)$?
10. (2 points) Suppose $\mathscr{I}(P) = T$, what is $v(\neg\neg P)$?
11. (2 points) Suppose $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = F$, what is $v(P \rightarrow \neg Q)$?
12. (2 points) Suppose $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = F$, what is $v(\neg P \wedge Q)$?
13. (2 points) Suppose $\mathscr{I}(P) = F$ and $\mathscr{I}(Q) = T$, what is $v(P \leftrightarrow \neg Q)$?
14. (2 points) $\mathscr{I}(P) = T$, $\mathscr{I}(R) = F$, and $\mathscr{I}(Q) = F$, determine the truth value of $(P \rightarrow \neg Q) \wedge R$
15. (2 points) Suppose $\mathscr{I}(P) = T, \mathscr{I}(Q) = F, \mathscr{I}(R) = F$, what is $v(P \vee \neg Q) \wedge \neg R)$?
16. (2 points) What is the truth value of $P \rightarrow P$?
17. (2 points) What is the truth value of $\neg P \wedge P$?

0.4 Truth-tree decomposition rules

Directions: Write the abbreviation (e.g. $\wedge D$) for the that should be used on the following wffs:

18. (2 points) $P \wedge Q$
19. (2 points) $P \vee Q$
20. (2 points) $P \rightarrow Q$
21. (2 points) $P \leftrightarrow Q$
22. (2 points) $\neg R \wedge R$
23. (2 points) $\neg(R \vee R)$
24. (2 points) $\neg(R \wedge R)$
25. (2 points) $\neg(R \vee R)$
26. (2 points) $\neg A \rightarrow B$
27. (2 points) $\neg(\neg R \leftrightarrow R)$
28. (2 points) $\neg P \vee \neg Q$?
29. (2 points) $\neg R \leftrightarrow \neg R$?
30. (2 points) $\neg(A \wedge \neg B)$?

0.5 Truth table and tree Construction

31. (10 points) On the answer sheet, construct a **truth table** for the following proposition and determine whether it is a contingency, tautology, or contradiction: $\neg(P \wedge Q) \leftrightarrow P$. To receive full credit, you must (i) construct the entire truth table, (ii) label whether it is a contingency, tautology, or contradiction, and (iii) clearly explain why the table shows the wff in question has the property you say it does.

32. (10 points) On the answer sheet, construct a **truth tree** for the following set of wffs and determine whether the set is consistent or inconsistent: $P \wedge \neg Q, \neg Q \rightarrow R, S \wedge \neg M$. To receive full credit, you must (i) construct the entire truth tree, (ii) label whether it is a consistent or inconsistent, and (iii) either demonstrate that the tree is consistent by recovering an interpretation (e.g. $\mathscr{I}(P) = T, \mathscr{I}(Q) = F$) from the tree *or* clearly explaining why the tree shows the set to be inconsistent.

33. (10 points) On the answer sheet, construct either a **truth table** or a **truth tree** for the following set of wffs: $\neg(P \vee Q), \neg(P \rightarrow Q)$. To receive full credit, you must (i) construct the entire truth table/tree, (ii) label whether it is a consistent or inconsistent, and (iii) if the set of wffs is consistent, write out the interpretation or identify the row (e.g. $\mathscr{I}(P) = T, \mathscr{I}(Q) = F$) demonstrating consistency.

34. (10 points) On the answer sheet, construct either a **truth table** or a **truth tree** for the following argument: $(P \vee S) \rightarrow \neg(Q \vee R) \models (\neg P \vee Z)$. To receive full credit, you must (i) construct the entire truth table or tree, (ii) label whether it is a valid or invalid (that is, entailment of non-entailment), and (iii) if the argument is invalid, identify the row or write out the interpretation (e.g. $\mathscr{I}(P) = T, \mathscr{I}(Q) = F$) demonstrating invalidity.

Congratulations!

- turn in your answer sheet at the front of the classroom,
- check to see if your homework has been graded,
- feel free to leave. Class is complete.
- answers to the exam will be posted to CANVAS.

$P \wedge Q$		$P \vee Q$	
P	$\wedge D$	P $\swarrow$ $\searrow$ Q	$\vee D$
Q	$\wedge D$		
$\neg(P \vee Q)$		$\neg(P \wedge Q)$	
$\neg(P)$	$\neg \vee D$	$\neg(P)$ $\swarrow$ $\searrow$ $\neg(Q)$	$\neg \wedge D$
$\neg(Q)$	$\neg \vee D$		
$\neg(P \to Q)$		$P \leftrightarrow Q$	
P	$\neg \to D$	P $\swarrow$ $\searrow$ $\neg(P)$	$\leftrightarrow D$
$\neg(Q)$	$\neg \to D$	Q $\qquad$ $\neg(Q)$	$\leftrightarrow D$
$\neg\neg(P)$		$\neg(P \leftrightarrow Q)$	
P	$\neg\neg D$	P $\swarrow$ $\searrow$ $\neg(P)$	$\neg \leftrightarrow D$
		$\neg(Q)$ $\qquad$ Q	$\neg \leftrightarrow D$
		$(P \to Q)$	
		$\neg(P)$ $\swarrow$ $\searrow$ Q	$\to D$

Table 1: Truth tree decomposition rules for **PL**

P	$\neg P$
T	F
F	T

Table 2: Truth Table: Negation

P	R	$P \wedge R$	$P \vee R$	$P \to R$	$P \leftrightarrow R$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Table 3: Truth Table: Conjunction, Disjunction, Conditional, and Biconditional

Directions: Remove this page from the exam. This page is your answer sheet. Please write your **name** on the top of the page and please write clearly. This page is specifically correlated to your exam so do not exchange your exam with your neighbor. **3Uv2x3**

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____
41. _____
42. _____
43 _____
44 _____
45 _____
46 _____
47 _____
48 _____
49 _____
50 _____