Directions: This exam has 30 questions, for a total of 100 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

1 Multiple Choice

Choose the best answer.

1. (2 points) An interpretation of **RL** is a function that does what (indicate all that apply):
   A. specifies what objects are in the domain.
   B. assigns truth values to n-place predicate terms followed by n terms.
   C. for each name in **RL** it assigns that name one and only one item in $\mathcal{D}$
   D. for each n-place predicate term in **RL**assigns, it assigns that predicate term a set of n-tuples composed of elements from $\mathcal{D}$
   E. assigns truth values to objects and wffs

2. (2 points) What is the principal weakness of **PL** in comparison to **RL**
   A. **PL** is not expressive enough: there are valid English arguments that can be expressed in **RL** that cannot be expressed in **PL**
   B. **PL** is too expressive: there are valid arguments in **PL** for which it would be impossible to express in English.
   C. **PL** has an imprecise syntax, while the syntax of **RL** is fully precise.
   D. **PL** has an imprecise semantics, while the semantics of **RL** is fully precise.

3. (2 points) What is a model ($\mathcal{D}$)?
   A. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and an interpretation function ($\mathscr{I}$)
   B. a model ($\mathcal{M}$) is a three-part structure consisting of a domain ($\mathcal{D}$), an interpretation function ($\mathscr{I}$), and a valuation (v) function.
   C. a model ($\mathcal{M}$) is a two-part structure consisting of a domain ($\mathcal{D}$) and a valuation function v where the valuation function assigns truth values to RL-wffs.
   D. a model ($\mathcal{M}$) is a single-part structure consisting of a domain ($\mathcal{D}$)

4. (2 points) In a predicate logic tree, under what conditions is a branch that contains a universally quantified wff (e.g. $(\forall x)Px$) considered a *completed open branch* (indicate all that apply)
   A. when $(\forall x)Px$ has been decomposed into $\neg(\exists x)Px$
   B. when $\Diamond P$ has been decomposed and relativized to a possible world, e.g., irj
   C. when $(\forall x)Px$ has been decomposed for every name $a, b, c, \ldots$ that occurs in that branch
   D. when all the complex wffs (non-literals) that are in that branch and that can be decomposed have been decomposed
   E. when the branch is not closed, viz., does not contain a wff and its literal negation

5. (2 points) What is a deductive apparatus for **RL**?
   A. a set of rules of derivation that express which wffs ϕ can be written after which wffs ψ in a derivation.
   B. a set of rules that state how a tree is supposed to look, e.g. horizontally rather than vertically.

C. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.

D. a set of rules that state that the rows in a proof need to be numbered.

E. a way of listing off each wff one right after another

6. (2 points) What is a derivation of **Q** using **RD**?

   A. a finite string of formulas from a set Γ of **RL** wffs where (i) the last formula in the string is Q and (ii) each formula is either a premise, an assumption, or is the result of the preceding formulas and the deductive apparatus.

   B. finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with $\mathbf{Q}$.

   C. a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with $\mathbf{Q}$.

   D. an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with $\mathbf{Q}$.

1.1 Symbols

7. (2 points) Which of the following symbols are **RL** names (indicate all that apply)?

   A. b

   B. y

   C. $\exists$

   D. m

   E. n

   F. $\forall$

   G. $\diamond$

1.2 Syntax

State whether the following formulas are wffs. You can assume that H is a one-place predicate, that L is a two-place predicate, and conventions for simplifying wffs are present.

8. (2 points) Hab

9. (2 points) $Ha \vee \neg Hb$

10. (2 points) $(\forall x)Hx$

11. (2 points) $(\exists y)(\forall x)(Lxy \wedge \neg Hx)$

1.3 Semantics

Directions: Determine whether the following wffs are true or false by using the following model: $\mathcal{D} = \{1,2,3,4,5\}$, $\mathscr{I}(a) = 1$, $\mathscr{I}(b) = 2$, $\mathscr{I}(c) = 3$, $\mathscr{I}(d) = 4$, $\mathscr{I}(e) = 5$, for all other names α, $\mathscr{I}(\alpha) = 4$, $\mathscr{I}(N) = \{1,2,3,4,5\}$, $\mathscr{I}(G) = \{\langle 2,1\rangle, \langle 3,2\rangle, \langle 3,1\rangle, \langle 5,1\rangle\}$, $\mathscr{I}(I) = \{\}$, $\mathscr{I}(E) = \{2,4\}$, $\mathscr{I}(O) = \{1,3,5\}$

12. (2 points) Oc

13. (2 points) $(\forall x)Nx$

14. (2 points) $(\exists y)Iy$

15. (2 points) $(\forall x)(Ex \wedge Ox)$

16. (2 points) Gab

17. (2 points) $(\exists x)(Gex)$

1.4 Translation

Directions: Translate the following English sentences into the language of predicate logic. Write the formula on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a) =$ Ava, $\mathscr{I}(j) =$ Jon, $\mathscr{I}(e) =$ Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

18. (2 points) Ava is not happy

19. (2 points) Ava loves Jon.

20. (2 points) Someone is *both* rich and happy.

21. (2 points) Someone is rich and someone is happy.

22. (2 points) All happy people are rich.

Directions: Translate the following predicate logic wffs into English. Write your translation on the line provided. Use the following translation key as your guide: $\mathcal{D}$=people, $\mathscr{I}(a) =$ Ava, $\mathscr{I}(j) =$ Jon, $\mathscr{I}(e) =$ Eve, $\mathscr{I}(Lxy) = x$ loves y, $\mathscr{I}(Hx) = x$ is happy. $\mathscr{I}(Rx) = x$ is rich.

23. (2 points) $(\forall x)Lxx$

24. (2 points) $(\forall x)Lxa$

25. (2 points) $(\forall x)(Hx \rightarrow Lxx)$

2 Trees and Proofs

Directions: Use a truth-tree to determine whether the following sets of wffs are consistent/inconsistent or arguments are valid/invalid. If the tree shows the set to be consistent or the argument to be invalid, construct a model illustrating this fact. (Rubric: Tree=5pts, Property=1pt, Model=4pts, if applicable)

26. (10 points) Determine consistent/inconsistent: $Pa, Qb, (\exists x)\neg Px, (\forall x)Qx$

27. (10 points) Determine semantic entailment: $(\exists x)Px, (\exists x)Mx \vdash (\forall x)(Px \rightarrow Mx)$

Directions: Solve the following proofs.

28. (10 points) $(\forall x)Bx \vdash (\exists x)Bx$

29. (10 points) $Lab, (\forall x)Bxx \vdash (\forall y)Byy$

30. (10 points) $Pa, (\exists x)(Ax \wedge Bx) \vdash (\exists x)Ax$

$P \wedge Q$ $P \quad \wedge D$ $Q \quad \wedge D$	$P \vee Q$ $P \swarrow \quad \searrow Q \quad \vee D$
$\neg(P \vee Q)$ $\neg(P) \quad \neg \vee D$ $\neg(Q) \quad \neg \vee D$	$\neg(P \wedge Q)$ $\neg(P) \swarrow \quad \searrow \neg(Q) \quad \neg \wedge D$
$\neg(P \rightarrow Q)$ $P \quad \neg \rightarrow D$ $\neg(Q) \quad \neg \rightarrow D$	$(P \rightarrow Q)$ $\neg(P) \swarrow \quad \searrow Q \quad \rightarrow D$
$P \leftrightarrow Q$ $P \swarrow \quad \searrow \neg(P) \quad \leftrightarrow D$ $Q \quad \quad \neg(Q) \quad \leftrightarrow D$	$\neg\neg(P)$ $P \quad \neg\neg D$
$\neg(P \leftrightarrow Q)$ $P \swarrow \quad \searrow \neg(P) \quad \neg \leftrightarrow D$ $\neg(Q) \quad \quad Q \quad \neg \leftrightarrow D$	
$\neg(\exists x)\phi$✓ $(\forall x)\neg(\phi), \neg\exists D$	$\neg(\forall x)\phi$✓ $(\exists x)\neg(\phi), \neg\forall D$
$(\exists x)\phi$✓ $\phi(\alpha/x), \exists D$	$(\forall x)\phi$ $\phi(\alpha/x), \forall D$

Table 1: Truth tree decomposition rules for **PL** and **RL**

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$ or $P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction ($\rightarrow I$)

n	$\quad$ | P	A
$\vdots$	$\quad$ | $\vdots$	
$(n+1)$	$\quad$ | Q	
$(n+2)$	$P \rightarrow Q$	$\rightarrow I$, n–$(n+1)$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$\neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Derivation Rule – Negation Elimination ($\neg E$)

n	$\neg(P)$	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$\neg Q$	
$(n+3)$	P	$\neg E$, n–$(n+2)$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

1	$P \vee Q$	P
n	P	A
$\vdots$	$\vdots$	
$(n+1)$	R	
(i)	Q	A
$\vdots$	$\vdots$	
$(i+1)$	R	
(k)	R	$\vee E$, 1, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
(i)	Q	A
$\vdots$	$\vdots$	
$(i+1)$	P	
(k)	$P \leftrightarrow Q$	$\leftrightarrow I$, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \vee Q, \neg Q \vdash P$ or $P \vee Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \rightarrow Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$

Derivation Rule – Implication (IMP)

$P \rightarrow Q \dashv\vdash \neg P \vee Q$

Derivation Rule – Universal Elimination ($\forall E$)

$(\forall x)\phi(x_1 \ldots x_n) \vdash \phi(\alpha_1 \ldots \alpha_n / x_1 \ldots x_n)$ where x is not in $\phi(\alpha_1 \ldots \alpha_n)$

Derivation Rule – Existential Introduction ($\exists I$)

$\phi(\alpha_i) \vdash (\exists x)\phi(x_n/\alpha_n)$ where x is not in $\phi(\alpha_i)$

Derivation Rule – Universal Introduction ($\forall I$)

$\phi(\alpha_1 \ldots \alpha_n) \vdash (\forall x)\phi(x_1 \ldots x_n/\alpha_1, \ldots \alpha_n)$ where the name α does not occur as premise, as an assumption in an open subproof, or in $(\forall x)\phi(x_1 \ldots x_n/\alpha_1, \ldots \alpha_n)$ and where x is not in $\phi(\alpha_1 \ldots \alpha_n)$

Derivation Rule – Existential Elimination ($\exists E$)

1	$(\exists x)\mathbf{P}$	P
n	$\mid\ \mathbf{P}(a/x)$	A
$\vdots$	$\mid\ \vdots$	
$(n+1)$	$\mid\ \mathbf{Q}$	
(k)	$\mathbf{Q}$	$\exists E$, 1, n–$(n+1)$

Derivation Rule – Quantifier Negation (QN)

$\neg(\forall x)P \dashv\vdash (\exists x)\neg P$ or $\neg(\exists x)P \dashv\vdash (\forall x)\neg P$