Vagueness, Language, and the problem of the heaps: Peirce's dissolution of the Sorites Paradox[1]

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In this paper,[2] I provide a Peircean solution to the sorites paradox.[3] This solution is "Peircean" rather than Peirce's because (1) Peirce did not offer a solution but (2) it is inspired by several claims made by Peirce.[4]

1. The Sorites Paradox

There are terms[5] for which there is indeterminacy concerning the limit of their application. These terms appear fuzzy around the edges and their fuzziness doesn't go away no matter how much we know about the objects they stand for or the qualities they signify. Vague terms are said to be "inquiry resistant" in that "no investigation of the facts will resolve the question of whether a predicate applies to its borderline cases" (Burns 1995:28-9). You might know precisely how much money someone has to the cent, but remain unsure whether you can call them "rich". As Sainsbury (1995:590) puts this same point, "[y]ou may know how tall someone is to the millimetre, yet be unable to say whether or not he is tall. You may see a shade under perfect conditions for assessing its colour, yet be unable to say whether or not it is red."

Terms that are indeterminate in this way are said to be *borderline vague* and such vagueness gives rise to a group of paradoxical arguments called "sorites arguments." There are several ways to express the paradox, but let's consider the three most popular versions. Let '$a_{\$n}$' stand for an individual *a* with a number *n* of dollars.

The Many-Conditionals Sorites

P1	$a_{\$250K}$ is rich.
P2	If $a_{\$250K}$ is a rich, then $a_{\$249K}$ is a rich.
P3	If $a_{\$249K}$ is a rich, then $a_{\$248K}$ is a rich.
…	
Pn	If $a_{\$1K}$ is a rich, then $a_{\$0K}$ is a rich.
C	Therefore, $a_{\$0K}$ is a rich.

Another way to express the sorites argument is with a universally quantified premise (**P2**).

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[3] See (Hyde 2011:15-16) for an overview of whether being sorites-susceptible is necessary for vagueness.

[4] Abbreviations of references to Peirce's work follow these conventions: CP#.# = (Peirce 1960); EP2:# = (Peirce 1998); HL:# = (Peirce 1997); R#:# & L#:# = (Peirce 1963-1970); W#.# = (Peirce 1982-2010). For rejected manuscript pages, an 'x' is placed after the manuscript page number, e.g. R343:32x.

[5] For vagueness in aesthetics, see (Tiercelin 2005) & (Gilmore 2009), metaphysics (Lane 2003:70), in the context of abortion debates (Levin 1985; English 1975:236). For an overview of vagueness in law, see (Endicott 2011). See also, (Waldron 1994, 2002; Poirier 2002; Endicott 2000, 1997; for an example, see Rurka 1999)

THE UNIVERSALLY QUANTIFIED PREMISE SORITES

P1 $a_{\$250K}$ is rich.
P2 For any number *n*, if $a_{\$n}$ is a rich, then $a_{\$(n-1)}$ is a rich.
C Therefore, $a_{\$0}$ is a rich.

A third way of thinking about the paradox is to put individuals in an ordered series according to the amount of money they have. Starting from the richest individual, we ask an interpreter "is *a* rich?" If the interpreter says "yes", then we move to an individual with one less dollar, and ask the same question. We keep repeating this process until either the interpreter says that a person with $\$_{1K}$ has a rich salary or until they decide to draw a line between the rich and not rich at some particular dollar amount.

FORCED MARCH SORITES

Definitely not rich	**Borderline Cases**	**Definitely rich**
$1K	$n	$250K
←		
is a rich?	*is a rich?*	*is a rich?*

Sorites arguments are *paradoxical* in that from seemingly true premises and by uncontroversial reasoning a seemingly false conclusion follows. Arguments of the above type are considered a species of the sorites paradox provided at least three conditions hold.

First, the minor premise (or its equivalent) must appear true. For example, P1 in the Quantified Premise version of the argument ("$a_{\$250K}$ is rich") appears true. If "$a_{\$250K}$ is rich" does not appear true, then increase the dollar amount until the minor premise finally appears true, e.g. "$a_{\$10M}$ is rich." Some individuals are skeptical whether people can be rich *simpliciter*. They contend that there is no such thing as *being rich* without specifying a context. To meet this condition, one can specify the context, e.g. "rich for 40 year-old woman with no student debt who is a partner at the most prestigious law firm in 2014."

Second, the adjacent items in the series—e.g. $a_{\$250K}$ and $a_{\$249K}$—must appear indiscriminable with respect to the application of the vague term. In our example, there is nothing about "rich" that would make it such that one would apply it to $a_{\$250K}$ but not apply it to the next item in the series ($a_{\$249K}$). That is, the difference does not seem large enough to warrant saying one member in the series is "rich" but the next one is not. If the difference is large enough to warrant such an difference in application, then make the difference between adjacent items in the series smaller. For example, rather than 10K, make it 1K, or make it $1.

Third, the argument must rely on some uncontroversial form of reasoning such as modus ponens or quantifier elimination and modus pones.

Obvious solutions to the paradox pose problems. First, if we *accept* the sorites argument, then natural language is incoherent. Since sorites arguments can be run both ways, accepting the sorites means that people with billions of dollars are rich and not rich. Second, denying the minor premise (**P1**) entails the implausible view that people with billions of dollars are not rich or that someone with $1 is rich. Finally, denying the major premise (**P2**) entails that when we use "rich", the term picks out a precise dollar amount that separates the *rich* and the *not rich*. This is controversial for at least two

reasons. First, there does appear to be anything in nature that separates the rich from the non-rich, and so our use of the term "rich" doesn't latch onto some property in the natural world that cleanly separates the rich from the non-rich. Second, denying the major premise severs the relation between *use* and *meaning* for while we don't use vague terms as though they have a sharp cut-off, they nevertheless signify one if the major premise is denied.

2. Scope & Qualifications

A complete theory of vagueness might do all of the following:

(G1) specify what vagueness is or how it should be conceived,
(G2) explain what borderline cases are,
(G3) solve the sorites paradox,
(G4) explain why the sorites paradox is compelling,
(G5) determine whether objects and properties are really vague,
(G6) provide a historical account of the development of vagueness,
(G7) offer a logic and complete semantic theory of vagueness,[6] and
(G8) generate a list of all different kinds of terms that are vague, e.g. adjectives, verbs, proper names, demonstratives, quantifiers, etc.

My goal here is *not* to give a complete theory of vagueness (G1-G8), but to offer a Peircean solution to the sorites paradox (G3).[7] To this end, it is necessary to make four further qualifications.

First, Peirce used the term "vague" in a variety of different ways.[8] This has led some to object that Peirce's use of the term was so multifarious and divorced from the contemporary notion of borderline vagueness that there can be no Peircean solution to the sorites paradox (Williamson 1994:52; cf. Agler 2013:203-4). While it is true that Peirce used "vague" in different ways, this does not mean that he had no conception of borderline vagueness. Elsewhere, I've argued that Peirce carved out a notion of borderline vagueness that is distinct from other forms of indeterminacy, e.g. generality, underspecificity, ambiguity, uninformativeness and so forth (Agler 2010, 2013). Since Peirce had a conception of borderline vagueness and the sorites paradox involves this type of vagueness, a Peircean solution to the sorites paradox is viable.

Second, Peirce does not appear to have tried to solve the sorites paradox.[9] My goal here then is not to try and give "Peirce's solution to the paradox", but as Peirce was aware of the sorites paradox as he

[6] Peirce famously claimed that to have worked out a logic of vagueness, but precisely what this is has been a subject of debate. Tiercelin, for example, claims that by "logic of vagueness" Peirce meant "significs" in the form of spelling out the different forms of indeterminacy and determinacy in relation to breadth (reference, denotation, extension) and depth (meaning, connotation, or intension). But this claim is problematic for Peirce claims to have worked out a "Stechiology (or Stoicheiology), Critic, and Methodeutic of the subject" (CP7.446). See (Shapiro 2011) for an overview of the relationship of logic to vagueness.

[7] This is not to say that the other aspects of a theory of vagueness are not important or that a solution to the sorites paradox is the most important aspect of a theory of vagueness. It is just that I think that it is probably best done by the community of inquirers and this community involves psychologists, lexicographers, linguistically-minded philosophers, linguists, metaphysicians, and perhaps those working in cognitive science, et alia.

[8] For some definitions, see (R283:138-139x, 1905; CP5.447, 5.505; R290:6-7, 291). For a discussion of Peirce's different ways of using the term, see (Brock 1969:114-115).

[9] Peirce uses the word "sorites" four times in the *Collected Papers* but these uses do not refer to the paradox but instead to a chain of syllogisms with suppressed premises (see CP2.449n, CP3.641, CP4.45, CP4.427).

mentions it in at least two places,[10] my goal is to use some resources in Peirce to try and give a *Peircean* solution to the sorites paradox.

Third, a great deal of the discussion of Peirce's account of vagueness addresses what he meant by the principle of non-contradiction. This is because Peirce says that something is *vague* if and only if the principle of non-contradiction does not apply.[11] The non-application of the principle of non-contradiction won't play a role in my solution and so I won't discuss it in this paper (Lane 1997; see 1999:285-291).

Fourth, there is discussion by Peirce of vagueness in the context of his pragmatism,[12] realism,[13] critical common-sensism,[14] and metaphysics.[15] I don't plan on addressing these topics either.

3. Four Peircean Theses

My sketch of a Peircean solution to the sorites paradox begins by putting forward **four** theses grounded in Peirce's writings.

(T1): When dealing with vague terms, we are dealing with signs whose meaning is established by convention.[16] Conventional signs do not signify their objects by being existentially-related to them (e.g. as smoke is a sign of fire)[17] or by resembling it (e.g. a straight line on a map resembles the straightness

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[10] The first is in a 27 October 1892 review of Alfred Sidgwick's (1892) *Distinction and Criticism of Belief*. There Peirce claims that ambiguity is a "confusion between ideas quite distinct, such as the unlimited and the immeasureable" but "[v]agueness is an indeterminacy in the limits of the application of an idea, as to how many grains of sand are required to make a heap, and the like." (27 October 1892, *The Nation*, p.167). A second place is in R283:137-138. After distinguishing ambiguity from vagueness, Peirce writes that "The old question of the "sorites," How many grains of sand are required to make a "heap," or "whole lot," is an instance" of vagueness (*R*283: 138x, 1905).

[11] (Brock 1969; Hookway 2002; Brock 1979; Lane 1997).

[12] According to Robert Almeder (1983:330), if the pragmatic maxim specifies the meaning of a sentence by translating it into a set of conditional statements where the *antecedents* specify some operation to be performed and the *consequents* specify the observable phenomena that should result, then our understanding of the meaning of any of sentence is *partial*. It is *partial* because the conditionals that give the meaning of a sentence are *general descriptions* and no general description is capable of fully specifying an absolutely determinate object.

[13] See (Tiercelin 1992).

[14] With respect to critical common-sense, Peirce takes certain indubitable propositions to be invariably vague and claims to have an a priori and an experimental proof to show this. I don't plan on addressing these aspects of Peirce's theory here.

[15] In trying to separate Peirce's account of the semiotics of vagueness from ontology, Tiercelin (2005:233, 240) might object to this as she argues that, for Peirce, "logic is inseparable from *ontology*; therefore, any account of vagueness will never be dealt with as such" and that Peirce's "logical project is thus inseparable from an ontological account of vagueness and from an elaboration of a realistic (though not Platonist) position which takes vagueness as one of its central tenets."

[16] (cf. CP2.297, CP2.307)

[17] Concerning the claim that conventional signs are not indices, Peirce writes "Take, for example, the word "man." These three letters are not in the least like a man; nor is the sound with which they are associated. Neither is the word existentially connected with any man as an index. It cannot be so, since the word is not an existence at all. The word does not consist of three films of ink. If the word "man" occurs hundreds of times in a book of which myriads of copies are printed, all those millions of triplets of patches of ink are embodiments of one and the same word. I call each of those embodiments a replica of the symbol. This shows that the word is

of a road).[18] Rather, Peirce says that conventional signs are *generals* (types)[19] that stand for the things that they do and signify the qualities that they do solely because there is a habit of associating the term with those things/qualities.[20] The particular habit that allows for particular encounters with the term (tokens or replicas) to signify one thing rather than another is, Peirce writes, "arbitrary" (CP3.360) but yet "it has been agreed, [the sign] shall be significant" (CP2.246).[21]

The totality of the predicates of a sign (or qualities it signifies) refers to what Peirce calls the sign's "depth" (he also uses the terms "comprehension," "content," "force," "connotation," "sense," "intension"). All of the conceivable qualities predicated of a term in its definition is what Peirce calls the sign's "essential depth" (we might call this a sign's "conventional intension").[22] The essential depth of a term like "rich" is not simply its dictionary definition. For example, one dictionary entry for "rich" is "having a lot of money or possessions", but in referring to the *conceivable* qualities predicated of a term, the essential depth of "rich" also includes those qualities that we *would* predicate of "rich" (see W2:75). So, even though we don't actually predicate "having over $10M" and "having over $15M" to rich, they are virtually predicated given our habit of using this term.[23] In saying that the essential depth involves the conceivable qualities referenced by a *dictionary*, Peirce is, I think, referring to what the term signifies through common usage or collective competence.[24] As such, "rich" also has a "banal signification", i.e., the part that "goes without saying" or what is "known to every one" (CP2.432).

Thus, given that vague terms are not fully devoid of meaning, Peirce would have likely accepted the minor premise and rejected the conclusion of the sorites paradox. And, Peirce says as much when he writes that "[s]ome things are definitely much: others are definitely little" (R530:13, 2nd pagination).

(T2): While vague terms like rich have essential depth, Peirce says that they lack *fullness* in depth (CD entry). Peirce writes that vagueness is due to the "sign's not sufficiently expressing itself to allow of an

not a thing" (CP4.447).

[18] For example, the word "bird" does not point us to a particular bird (CP2.298) nor does it resemble a bird in any significant way.

[19] For example, Peirce says "it is itself a kind and not a single thing" (EP2:9).

[20] (see CP2.92; CP4.447; EP2:163; EP2:274: EP2:292)

[21] In other words, particular instances of the term "man" "become a sign only in the fact that a habit, or acquired law, will cause replicas [instances] of it to be interpreted as meaning man" (CP2.292). Elsewhere, concerning the term "man", Peirce writes "What is its nature? It consists in the really working general rule that three such patches seen by a person who knows English will effect his conduct and thoughts according to a rule" (CP4.447). In addition, Peirce writes that conventional signs are "*applicable to whatever may be found to realize the idea connected with the word*" (EP2:9).

[22] CP2.410. In contrast, the logical depth of a sign refers to the "totality of the predicates of a sign, and also the totality of the characters it signifies" (EP2:305, EP2:473-4, EP2:797-8). The importance of referring to the essential depth rather than other forms of depth is that the key issue concerns the vagueness of a sign's meaning and not an indeterminacy due to a lack of information on the part of the utterer. For example, when Peirce writes that "[i]t is only when the utterer talks of something with which the interpreter is not familiar that his talk becomes vague", this lack of familiarity may be due to an indeterminacy in terms of a familiarity with the object indicated by a sign or an indeterminacy due to what the sign is to mean. Suppose, Peirce writes, "that he [the speaker] syas, "A man I knew had blue hair." He refers to the man vaguely, because the interpreter did not know him, or did not know him well,—had not seen him; for if he had, no information would be communicated, and the proposition would not actually function as a proposition" (R530:17).

[23] "The meaning of a word is more fully the sum total of all the conditional predictions which the person who uses it intends to make himself responsible for or intends to deny. That conscious or quasi conscious intention in using the word is the second grade of meaning." (from *What Makes a REasoning sound or Meaning)*

[24] (cf. Nadin 1983:160)

indubitable determinate interpretation" (CP5.448n1). The particular insufficiency in question pertains to the sign's intension (depth) as the sign does not specify a sharp quality that could serve as a criterion to determine whether a borderline case falls within its extension, e.g. *only those having over $10M*. Without a precise intension, there is uncertainty about whether certain borderline cases of rich are rich or not.[25]

For Peirce, it isn't the case that the sign signifies something precise and that we fail to know precisely what it signifies.[26] Rather, Peirce says that vague terms have "no definite meaning."[27] Even if we had perfect knowledge of a language user's mind, it is "intrinsically uncertain" whether a particular use of "red" would apply to a borderline case of red,[28] i.e., a speaker's use of "red" leaves it *uncertain* whether "red" would apply to shades of hair between *red* and *sandy* (see R48).[29]

[25] Concerning the vagueness of colors and numbers, Peirce writes "One may entertain the theory that all vagueness is due to a defect of cogitation or cognition. It is a natural kind of nominalism the justice of which it would be remote from the purpose of this analysis to consider. The vagueness of characters is of different kinds. The quality of redness and the quality of blueness differ without differing in any essential character which one has but the other lacks. The otherness of them is as irrational as the qualities themselves, if not more so. It appears to consist in a mutual war between them, in our taste. But the characters of integers are not of this irrational kind. In another regard, however, they are vague. Thus we say that the two characters of 4, of being the sum of 2 and 2, and of being the product of ~ and 2, are different characters" (CP4.344)

[26] Nadin writes that "[v]agueness concerns informed depth." For a criticism of this claim, see (Agler 2010:76-80).

[27] To the question whether a certain newly found skeleton was the skeleton of a man rather than an anthropoïd ape, the reply "Yes and no" might, in a certain sense, be justifiable. Namely, owing to our conception of what a man is having been formed without thinking of the possibility of such a creature as to which this skeleton belongs, the question really has no definite meaning (R 596: 16).

[28] (Peirce 1902:748).

[29] Peirce contends that if we were to say "A certain friend of mine has only a hundred and twenty-three hairs on his pate at most", this sentence is *indeterminate* but *not vague* in the sense that there is a number of things that are neither affirmed nor denied, e.g. that sentence doesn't say what color hair his friend has nor does he assert what arrangement the hairs are in. But, that kind of lack of indeterminateness is not vagueness because there is "no doubt what I mean to assert about the person in question" (R48:7-8).

In rejecting that a vague term has a definite meaning,[30] "rich" does not signify a sharp-cut off that sharply divides those who are rich from those who are not rich. Thus, Peirce would have rejected the **epistemic theory of vagueness** which holds that our use of the predicate "rich" has a definite intension (it indicates a sharp cut-off property) that sharply separates those that are rich from those that are not rich (cf. Sorensen 1988, 2001; Williamson 1992, 1994; Graff 2000). For the epistemic theory, vagueness is simply the result of our inability to know where that cut-off determined by rich is.

In short, while **T1** contends that a vague term like "rich" signifies a set of qualities that allow us to say some things are definitely rich and others are definitely not rich, there is no corresponding habit of interpretation (convention) that signifies whether the borderline cases are rich or not rich.

FIRST-STAGE IN THE PEIRCEAN APPROACH

Definitely not rich	**Borderline Cases**	**Definitely rich**
"$10K is rich" is false	?	"$10M is rich" is true
	I^p	*unique partial interpretation*
	↑	
	"rich"	

(T3): Peirce contends that our vague terms can signify different senses, that we tolerate different uses of vague terms, and that the vagueness or precision of a sign is determined relative to a purpose.

[30] Peirce rejected the claim that a term could be made fully precise, but this is another matter. With respect to the impossibility of absolute precision, Peirce writes:

> "In reference to the doctrine of individuals, two distinctions should be borne in mind. The logical atom, or term not capable of logical division, must be one of which every predicate may be universally affirmed or denied. For, let A be such a term. Then, if it is neither true that all A is X nor that no A is X, it must be true that some A is X and some A is not X; and therefore A may be divided into A that is X and A that is not X, which is contrary to its nature as a logical atom. Such a term can be realized neither in thought nor in sense. Not in sense, because our organs of sense are special -- the eye, for example, not immediately informing us of taste, so that an image on the retina is indeterminate in respect to sweetness and non-sweetness. When I see a thing, I do not see that it is not sweet, nor do I see that it is sweet; and therefore what I see is capable of logical division into the sweet and the not sweet. It is customary to assume that visual images are absolutely determinate in respect to color, but even this may be doubted. I know no facts which prove that there is never the least vagueness in the immediate sensation. In thought, an absolutely determinate term cannot be realized, because, not being given by sense, such a concept would have to be formed by synthesis, and there would be no end to the synthesis because there is no limit to the number of possible predicates. A logical atom, then, like a point in space, would involve for its precise determination an endless process. We can only say, in a general way, that a term, however determinate, may be made more determinate still, but not that it can be made absolutely determinate. Such a term as "the second Philip of Macedon" is still capable of logical division--into Philip drunk and Philip sober, for example; but we call it individual because that which is denoted by it is in only one place at one time. It is a term not absolutely indivisible, but indivisible as long as we neglect differences of time and the differences which accompany them. Such differences we habitually disregard in the logical division of substances. In the division of relations, etc., we do not, of course, disregard these differences, but we disregard some others" (CP3.93; DNLR)

First, Peirce adopts what I call the **flexibility thesis.** In several places, Peirce suggests that for a single borderline case **b** of a vague predicate **P**, a speaker **S** may at one point in time say "**b** is **P**" and at another point in time say "**b** is not **P**" without contradiction. In other words, we may decide that a borderline case **b** is rich, but later then decide that **b** is not "rich".[31] In his *Baldwin Dictionary* entry for "vague", Peirce writes that language users change their minds about whether borderline cases should be asserted or denied of a vague term. Peirce writes: "one day he would regard the proposition as excluding, another as admitting, those states of things" (Peirce 1902:748). Elsewhere he says that "there are shades of hair between sandy and red which I might one day call red while on another day I might say, "No, that is reddish, but not red"" (R48:8-10). And again, between the cases that are definitely much and definitely little there "is a border to which a man might at one hour apply the word "much," at another the word "little," in the endeavor to express the same truth" (R530: 13, 2nd pag., c.1903).[32]

Second, Peirce accepts what I call the **tolerance thesis.** For a borderline case **b** of a vague predicate **P**, a speaker $\mathbf{S_1}$ may truthfully assert "**b** is **P**" while another speaker $\mathbf{S_2}$ may truthfully assert "**b** is not **P**" without either $\mathbf{S_1}$ or $\mathbf{S_2}$ saying something false. In other words, the **tolerance thesis** contends that different language users can use a vague term like "rich" in different ways with respect to borderline cases. Peirce writes,

> That is 'much' which a man thoroughly well acquainted with the force of words might call much in the endeavor to express the truth; and of a person who calls such a thing 'much', must be admitted that he speaks the truth, even though the same thing might also be called little, that is, *not* much, in an endeavor by a perfect master of the language to speak the truth (R530: 14).[33]

The flexibility and tolerance theses conjointly imply that there is no single, objective sharpening of a vague term that settles whether it does or does not apply to borderline cases. There is, instead, a range of permissibility or a range of equally acceptable interpretations for these terms.

Third, Peirce claims that the classification of "vague" and "precise" are defined relative to some *purpose* shared by the speaker and listener. I call this the **purpose-relative thesis**. Peirce writes that it is a blunder to think that that a sign "is indeterminate simply because there is much to which it makes

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[31] It is worthwhile to point out that we are dealing with borderline cases of a predicate term and not the status of a boundary line on a continuous series, e.g. an instant in time or a point between an ink blot and a white piece of paper. Lane (1999) has argued that Peirce took these "boundary-propositions" to motivate his experiments in three-valued logic.

[32] Peirce (1902:748) writes that a proposition is "vague" when "there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed." He goes on to state that a consequence of this intrinsic uncertainty of the language user's habits is that "one day he would regard the proposition as excluding, another as admitting, those states of things" (Peirce 1902:748).

[33] Given this permissible variation, Peirce says that the same sentence "x is much" may be "both asserted and denied with truth" (R530:14). Elsewhere he notes that unless a proposition is made determinate "it may be true that a proposition is true and that a proposition is false" (EP2:351, 1905). The idea here, however, is not that there are true contradictions, but instead that a proposition may come out as true on one interpretation (and so may be asserted) and false on another interpretation (and so may be denied).

no reference" (CP4.448n1; cf. CP3.93).[34] As an example, Peirce (see C5.448n1) considers two Englishmen on a train. He says that even after they've identified Charles II of England (1630–1685) as their subject of conversation, it is possible that their subject might be made more determinate, e.g. they could isolate their conversation to Charles II as a boy, Charles II in exile, or Charles II as a monarch. But Peirce writes the two Englishmen have

> no purpose of splitting hairs in their talk; and the latitude of interpretation which constitutes the indeterminacy of a sign must be understood as a latitude which might affect the achievement of a purpose. For two signs whose meanings are for all possible purposes equivalent are absolutely equivalent (CP5.448n1).[35]

The same can be said of predicates for Peirce notes that if "one says that Charles II had dark auburn hair, the other will understand him quite precisely enough for all their possible purposes; and it will be a determinate predication" (CP5.448n1; cf. CP3.93).

In short, terms need not be made *fully precise* to be *precise enough* for a given purpose. This purpose-relative feature of Peirce's theory explains three phenomena.

First, it explains one of the reasons why language users only use precise terms. For if a precise term offers no more practical benefit than a vague term, then there isn't any reason to prefer the precise term over the vague one. Second, it explains why the initial reaction to the sorites paradox is one of *derision*. If you ask people whether the meaning of "tall" picks out a height that would allow you to identify the shortest tall person or if "rich" picks out a precise quantity of cash that would allow you to identify the poorest rich person, they want to know *why* this matters. Without a specification of the

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[34] "Another advantage of this definition is that it saves us from the blunder of thinking that a sign is indeterminate simply because there is much to which it makes no reference; that, for example, to say, "C.S. Peirce wrote this article," is indeterminate because it does not say what the color of the ink used was, who made the ink, how old the father of the ink-maker [was] when his son was born, nor what the aspect of the planets was when that father was born. By making the definition turn upon the interpretation, all that is cut off." - C5.448n1; cf.3.93)

[35] "Every sign has a single object, though this single object may be a single set or a single continuum of objects. No general description can identify an object. But the common sense of the interpreter of the sign will assure him that the object must be one of a limited collection of objects. Suppose, for example, two Englishmen to meet in a continental railway carriage. The total number of subjects of which there is any appreciable probability that one will speak to the other perhaps does not exceed a million; and each will have perhaps half that million not far below the surface of consciousness, so that each unit of it is ready to suggest itself. If one mentions Charles the Second, the other need not consider what possible Charles the Second is meant. It is no doubt the English Charles Second. Charles the Second of England was quite a different man on different days; and it might be said that without further specification the subject is not identified. But the two Englishmen have no purpose of splitting hairs in their talk; and the latitude of interpretation which constitutes the indeterminacy of a sign must be understood as a latitude which might affect the achievement of a purpose. For two signs whose meanings are for all possible purposes equivalent are absolutely equivalent. This, to be sure, is rank pragmaticism; for a purpose is an affection of action.

What has been said of subjects is as true of predicates. Suppose the chat of our pair of Englishmen had fallen upon the color of Charles II's hair. Now that colors are seen quite differently by different retinas is known. That the chromatic sense is much more varied than it is positively known to be is quite likely. It is very unlikely that either of the travelers is trained to observe colors or is a master of their nomenclature. But if one says that Charles II had dark auburn hair, the other will understand him quite precisely enough for all their possible purposes; and it will be a determinate predication." (C5.448n1; cf.3.393)

purpose this additional precision would serve, the sorites paradox evokes derision and annoyance for it appear to engage in semantics for the sake of engaging in semantics. Third, it explains why we want more precision in certain contexts and why attitudes toward the paradox change when the purpose of the precision is specified. If you ask the artist about the fuzzy boundary between the terms "art" and "craft", or the dietician about borderline cases of "eating too much", or the athlete about whether a supplement is "therapeutic" or "performance-enhancing," they quickly recognize that precision with these terms makes a difference as to whether one is allowed entry into an art show, whether one should seek counseling for eating, or whether one will be banned from further competitions.

(T4): Let me summarize. Suppose we start from the intuition that sentences with vague terms get a *partial interpretation*. What this means is that our interpretation of "rich" signifies qualities that allow us to say that those who are definitely rich are rich and those that are definitely not rich are not rich. However, this partial interpretation does not specify whether the borderline cases are rich or not rich, and so sentences about these cases seem as though they fall into a truth-value gap, i.e. they would not get assigned a truth value T or F in the interpretation. How does Peirce deal with this gap?

Peirce's does not deal with this gap by (i) *asserting* that when we utter a vague term, we say something precise (e.g. epistemicism), (ii) *filling* in the gap with additional semantic values (e.g. multi-valued theories),[36] or (iii) *preserving* the gap by saying that borderline cases are not-determinable (e.g. supervaluationist theories).[37] All of these approaches treat the meaning of vague terms in a *synchronic*[38] way, i.e., as though the whole meaning of a vague term is present at the moment of utterance. All of these approaches ignore the capacity of language users to develop what they mean by through future determination of the signs they use.[39]

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[36] As Lane (1999) has convincingly argued, Peirce's three-valued logic only applies to propositions about breaches in continuity (see n1). The **multi-valued approach** says that the sentences that do not get assigned a value of T or F, should be assigned a third value I or that the various sentences that occupy the gap should be assigned a truth value to the degree to which they are true or false. A somewhat crude argument for this view is this: since the classical values T and F cannot capture the status of sentences about borderline cases, unless a third value (or n-value) is admitted, the logical behavior of such sentences will not be captured. Keefe's (2000:90) objection to this is that additional semantic values are not necessary as supervaluationism can capture the logic of vagueness without introducing new semantic values. For various multi-valued approaches to vagueness, see (Tye 1994; Machina 1976; Edgington 1997; Smith 2003, 2008).

[37] **Supervaluationism** (Fine 1975; Keefe 2000, 2008) begins with the dea that vague predicates do not have definite extension. Rather, there is a range of possible extensions and it is unsettled which *one* of these extensions is the predicate's extension. The different extensions correspond to the different ways in which a vague predicate could be made precise, e.g. "tall" can be given a definite extension as *only those over 6'6* or *only those over 6'7*. The supervaluationist's insight then is this: while there is no semantic criterion for choosing between those different extensions, truth conditions for sentences with vague terms can be given by universally quantifying over the different admissible ways of making a vague term precise. A sentence is *true* if it is true on all ways of making that vague term precise, *false* if false on all ways of making that vague term precise, and *neither true nor false* otherwise. See (Smith 2008:80-81) for a partial list of the criteria involved in making these terms precise. For a more detailed description of supervaluationism, see (Smith 2008:76-82).

[38] Perhaps another way to think of it is, as Catherine Legg does. She says that modern thinking has been dominated by the Cartesian idea that the meaning of a sign is *private, incorrigible, and fully determined by the intention of a speaker* (Legg 2013)

[39] Objections to the multi-valued theory and supervaluation theories can be found in (Keefe 2000; Smith 2008; Williamson 1994).

In contrast, Peirce treats the intension (depth) of vague terms in a *dynamic* way. That is, a vague term's intension not only depends upon the existing conventions governing that term but how language users would develop that term by making it more precise. Peirce signals this commitment when he writes that "[t]he determination of the meaning [of a sign] depends upon the choice of one party [the speaker or listener], the other must follow it, or they will be at cross purposes; and the sign will fail to function as such" (R11:2). There is thus a difference between the synchronic model that treats meaning as an object to be analyzed at a given time in a given context and Peirce's dynamic (or diachronic) model that treats meaning as undergoing development in the hands of language users.[40]

Most of the time, Peirce says that it is the *speaker* of the vague term who has the right (or responsibility) to make the vague term more precise. Peirce writes, that while a sign is "apt to represent many things, the option as to what single thing it shall be taken to represent may be reserved by the utterer of it to whom it naturally belongs" (R9:2-3).[41] However, in other places, Peirce downplays the speaker's role[42] and says that what is important is that the further development does not come from the interpreter. For example, Peirce writes that the vague sign "reserves for some *other sign* or *experience* the function of completing the determination" (CP5.505, my emphasis).[43] For

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[40] Perhaps another way of thinking about what Peirce thinks here is not the dynamic model but *a modal model.* For example, Peirce says that "[i]ndefiniteness essentially involves a *May-be*; 'a man' is one man alone; but it *may be* this, it *may be* that, it *may be* each in term. To May-be is very peculiar in that it implies to [sic] *May-not-be*." (R641: 24 – 24 ½). Given that Peirce associates vagueness with the mode of possibility, Peirce might instead be taken as claiming that what is signified by the initial vague use of a term are a number of possible ways of making that vague term precise. Peirce writes that "Any assertion is said to be made in the mode of possibility "if, and only if it is conceivable that the affirmation and the denail of that which it so asserts should be both at once *true*" (R678:27). Elsewhere he contends that "[N]o present actual thought (which is a mere feeling) has any meaning, any intellectual value; for this lies not in what is actually thought, but in what this thought may be connected with in representation by subsequent thoughts; so that the meaning of a thought is altogether something virtual" (Peirce 1868).

[41] In R530:14-15, Peirce writes "[s]uppose that after a person had said that something was much, reserving, of course, his natural right to understand 'much' in any sense the word would bear and that he might choose [...]." A further example of this pertains to the sentence "A man whom I could mention seems to be a little conceited." Concerning this sentence, Peirce writes: "The *suggestion* here is that the man in view is the person addressed; but the utterer does not authorize such an interpretation or *any* other application of what she says. She can still say, if she likes, that she does *not* mean the person addressed. Every utterance naturally leaves the right of further exposition in the utterer; and therefore, in so far as a sign is indeterminate, it is vague, unless it is expressly or by a well-understood convention rendered general"(EP2:351, 1905).

Although Peirce says that the right belongs to the utterer, earlier he notes that a vague sign is one that "reserves further determination to be made in some other conceivable sign, *or at least does not appoint the interpreter as its deputy in this offic*e" (EP2:351, 1905, my emphasis). I take this to imply that we can use other signs, e.g. from context, to further determine the vague sign in question. Given that the sign itself does not signify what precise sense is meant, Peirce says that the future determination of a term is not the interpreter's (CP4.461, 1903).

[42] This also holds true when, Peirce writes "when one holds converse with oneself; for the self which signifies is always other than the self to whom the thought is signified" (R530:16).

[43] He writes that the vague sign "reserves further determination to be made in some other conceivable sign, or at least does not appoint the interpreter deputy in this office" (EP2:351, 1905). Peirce writes, "If a sign allows the utterer a certain latitude of choice as to what his meaning may be; so that he may perhaps defend its applicability in several ways (at least, if not, that is because of a fact and not because of the character of the sign) then the sign may be said to be *vague*, or *non-definite*" (R10:2-3; cf. R11:2). Although Peirce says that the right belongs to the utterer, earlier he notes that a vague sign is one that "reserves further determination to be made in some other conceivable sign, *or at least does not appoint the interpreter as its deputy in this offic*e" (EP2:351,

the purpose of solving the paradox, I want to put aside this potential ambiguity and interpret Peirce as saying that *speakers* have the right to make their use of vague terms more precise.[44]

It seems intuitive that the speaker (rather than the interpreter) should have this role for it is s/he who said it (we commonly ask speakers "what did you mean by that"). Peirce writes that it "naturally belongs" to the utterer.[45] In addition, it also fits with Peirce's view of assertion[46] since, according to Peirce, when the speaker asserts a sentence with a vague term, s/he offers the interpreter a kind of guarantee,[47] namely that s/he can make the term precise in some acceptable way that would make the sentence true.[48]

Assuming then that existing habits of interpreting vague terms signify certain definite qualities, but that the speaker is free to further determine the intension of a sign where no convention is present, one way that a speaker can make a vague term precise is by specifying a cut-off.[49] That is, a speaker can specify that "rich" means "only those with over $1M." And, in this way, sentences about borderline cases are made true or false given the speaker's proposed specification.[50]

In saying this, it should be cautioned that this further determination is in a context for a particular purpose (purpose-relative) and need not apply to other contexts (flexibility thesis) or bind the rest of the linguistic community to that specification (tolerance thesis).[51]

THE PEIRCEAN APPROACH TO VAGUE SENTENCES: POST SPEAKER STIPULATION

Definitely not rich	**Borderline Cases**	**Definitely rich**
"$10K *is rich" is*	$n	*"$250k is rich" is true.*

1905, my emphasis). I take this to imply that we can use other signs, e.g. from context, to further determine the vague sign in question. Given that the sign itself does not signify what precise sense is meant, Peirce says that the future determination of a term is not the interpreter's (CP4.461, 1903).

[44] According to Lane (1997:695-6) "[i]mprecise predications are taken to be covert existential quantifications."

[45] (R9:2; R530:14-15, 16; cf. 5.447-8, 5.506, R284:21)

[46] Jared Brock (1969:102) and others (e.g. Hookway 1985:237-8) take Peirce's theory of selection to be rooted in Peirce's broader theory of quantification and assertion.

[47] Peirce takes both parties in the selection process to be "perfectly competent" (R515:25x)

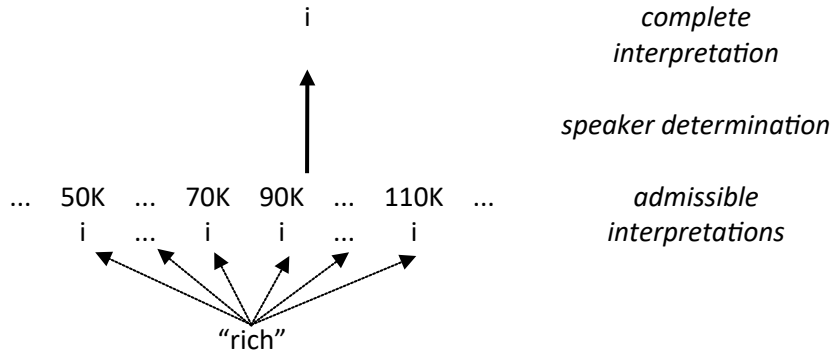
[48] (Brock 1969:105-6; cf. CP4.59, 1893; CP3.481; CP4.483)

[49] The choice of how to further make a vague term precise seems to be a *free* choice as Peirce says that the utterer has the "liberty of choice" (R515:20-21, cf. 25-26).

[50] Peirce's view thus bears some resemblance to the pragmatic theory of vagueness suggested by David Lewis and Linda Burns. For both Lewis and Burns, languages are precise abstract objects that are free from vagueness. Vagueness emerges because the linguistic conventions or habits of speakers don't select a specific language but a cluster of languages. As Lewis puts it, "languages themselves are free of vagueness but that the linguistic conventions of a population, or the linguistic habits of a person, select not a point but a fuzzy region in the space of precise languages" (Lewis, General Semantics, pp.64). In *Convention*, this makes the actual language we speak a kind of a resonance hybrid of the languages in the cluster (Convention, p.201). "Thus an ordinary, open-textured, imprecise language is a sort of blur of precise languages—a region not a point in the space of language" (Language and Languages, pp.35).

[51] Second, the manner in which a speaker makes a term precise is not by revealing the precise sense they *originally meant but did not convey* when they used a vague term. In other words, a speaker's decision to further determine a vague term is not analogous to laying one's cards on the table.[?] If the term is objectively vague, as opposed to simply unspecific or uninformative, the further determination of a term is a *new development* in the meaning of the sign. Finally, a speaker may never specify what s/he means by a vague term. In some cases, this isn't terribly problematic as a vague term is sufficiently precise in certain context, e.g. I'll take two spoonfuls of sugar with my coffee.

false.



4. A Peircean Solution to the Sorites Paradox

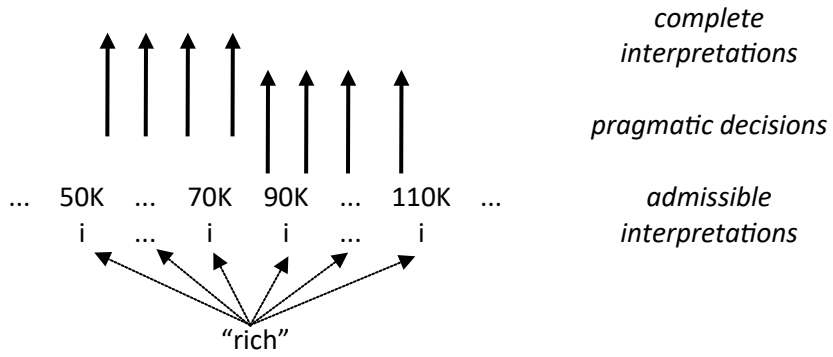
How might we solve the sorites paradox using the above four claims (T1)–(T4)? Recall the quantified premise of the sorites paradox:

> **QP:** For any number *n*, if $a_{\$n}$ is a rich, then $a_{\$(n-1)}$ is a rich.

Let's assume that the meaning of a vague term lies *not only* in what it signifies by convention *but also* the conceivable cut-off stipulated by a speaker. While no unique cut-off is signified by the vague term, we said that, if required, the speaker could make a vague term precise by specifying some cut-off. Even if we don't know how a speaker will make a vague term like "rich" precise, the sorites paradox can be solved. For while a single speaker may never actually stipulate a cut-off, and different speakers will make "rich" precise in different ways, on *every* further specification of a cut-off it will be the case that ***n* such that $a_{\$n}$ is a rich but $a_{\$(n-1)}$ is not a rich** (the denial of the QP). If it is part of the development of the meaning of every vague sign that it specify a cut-off, and *every stipulation of a cut-off* separates those that are rich from those that are not rich, then **QP** is false.

"Rich": Subject to Multiple Stipulations

Definitely not rich	**Borderline Cases**	**Definitely rich**
"$1K *is rich" is true.*	"$n is rich" is T or F	*"$250k is rich" is true.*



In other words, as every future conceivable determination of a vague term would make **QP** false, it makes *no practical difference* which one of these specifications the speaker chooses.[52] Even though the speaker may never specify a cut-off point for "rich" (as the term is precise enough given his/her purpose), if we take part of the meaning of "rich" to be found in the capacity of the speaker to specify a cut-off, then we can say that **QP** (or some conditional) is false.[53]

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[52] Similarly, If **QP** is false, then **not-QP (**there is a cut-off value between rich and not-rich) should be true. Notice that **not-QP** is true on every future determination of "rich". Although speakers may decide to make "rich" precise in different ways, i.e. draw the line between *rich* and *not rich* at different places, every future determination would posit a cut-off between rich and not-rich somewhere. Thus, each interpretation takes **not-QP** is **true**

[53] **So why do we get duped by the paradox?** I think that we get duped by the sorites paradox because we think that *denying the quantified premise* commits us to the actual existence of a sharp-cut off that is signified by our use of a vague term in a sentence. According to Peirce, this isn't the case. When we deny the quantified premise, what we are committed to is that we could choose a sharp-cut off if required. One does not yet exist but one of several different cut-offs could come into being given a future determination by a speaker. In other words, in the sorites paradox, we mistakenly reason from the fact that there is no stopping point signified (or even specifically intended) by our initial use of a term to the claim that the sign itself cannot signify a cut-off.

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