Conditionals and Necessary & Sufficient Conditions: An LSAT Primer

David W. Agler
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1. Necessary Conditions

To say that **A** is a *necessary condition* for **B**, is to say the following:

- without A, we won't have B.
- A is *required* to have B.
- The lack of A guarantees the lack of B
- B exists *only if* A exists

Examples

My car will run only if it has gas in it.

- Having gas in your car is a requirement for it to run. Without gas, the car won't run.

If you don't got the money, then you don't get the drugs.

- A drug dealer won't give you drugs unless you pay them money. Having money is a necessary requirement for getting drugs.

If Jane defriends Jon on Facebook, then Liz will defriend her.

- This one is a little trickier. It won't be the case that Jane defriends Jon unless Liz also defriends Jane.

Structures and terms that typically signal the *necessary* (N) condition:

Form	**Example**
if S *then* N	If my car starts, then there is gas in it.
S only if N	My car starts only if there is gas in it.
Every S is N	Every cat is a mammal.
Any S is N.	Any cat is a mammal.
N when S	There is gas in my car when it starts.
N whenever S	There is gas in my car whenever it starts.
S depends on N	My going to the dance depends on getting a date.
S only when N	My car starts only when there is gas in it.

2. Sufficient conditions

To say that *A* is a *sufficient condition* for *B,* is to say the following:

- if we have A, we will also have B.
- the presence of A guarantees the presence of B.
- the existence of A guarantees the existence of B
- if A exists, then B exists.

Examples

If my car starts, then there is gas in it.

- If my car starts, then I am guaranteed that there is gas in it.

If you have the money, then here are the drugs.

- Having the money is enough to get the drugs. It allows you to get the drugs if you want them.

If Jane defriends Jon on Facebook, then Liz will defriend her.

- Jane's defriending of Jon activates Liz's defriending of Jane.

Structures and terms that typically signal the *sufficient* (S) condition:

Form	**Example**
if S *then* N	If my car starts, then there is gas in it.
S only if N	My car starts only if there is gas in it.
Every S is N	Every cat is a mammal.
Any S is N.	Any cat is a mammal.
N when S	There is gas in my car when it starts.
N whenever S	There is gas in my car whenever it starts.
S depends on N	My going to the dance depends on getting a date.
S only when N	My car starts only when there is gas in it.
When S, I know N	When my car starts, I know that there is gas in it.

In thinking about necessary and sufficient conditions, there are **three** further ideas to keep in mind. First, A may be sufficient for B, but C, D, and/or E might also be sufficient for B. For example,

Getting an A in logic is sufficient for passing.

But, notice that while getting an A is sufficient for passing, so are getting a variety of other grades. For example,

Getting a B in logic is sufficient for passing.
Getting a C in logic is sufficient for passing.

Second, A may be necessary for B, but this does not mean that A is sufficient for B. For example,

My car will run only if it has gas in it, but *having gas doesn't guarantee it will run*.

Third, A may be both necessary and sufficient for B. Oftentimes this is signaled by the stronger "if and only if" phrase.

Jon will go to the dance if and only if he has a date.

What this says is that having a date is both necessary and sufficient for Jon's going to the dance. That is, if he has a date, then he will go to the dance AND he will go to the dance only if he has a date.

3. Conditionals

So what do necessary and sufficient conditions have to do with logic? Or, to ask this in a different way, how can logic help us in thinking about (or representing) necessary and sufficient conditions. To answer this question, consider that a *conditional* is a statement of the form "if **S**, then **P**". Conditionals consist of two parts: the antecedent and the consequent. In the "if **S**, then **P"** structure, "**S**" is the antecedent and "**P**" is the consequent.

If my car is running (antecedent), then there is gas in it (consequent)

In PL, we represent conditionals as "S→P". So, we can represent the above conditional as follows:

C→G

Next, let's consider the truth table for conditionals:

C	G	C→G
T	T	T
T	F	F
F	T	T
F	F	T

At first, let's just consider the cases where "C→G" is true.

C	G	C→G
T	T	T
F	T	T
F	F	T

Notice two things about *true* conditionals:

(1) on the condition that the antecedent is true, the consequent is true.

- That is, the truth of the antecedent is *sufficient* for the truth of the consequent.

In saying that the truth of the antecedent is sufficient for the truth of the consequent, what we are saying is that if a conditional and its antecedent are true, then we are guaranteed that the consequent is true.

S is sufficient for P		
if	S→P	is true
and	S	is true
then	P	is guaranteed true

(2) the antecedent is never true unless the consequent is also true.

- That is, the truth of the consequent is *necessary* for the truth of the antecedent. A true consequent does not guarantee a true antecedent, but the antecedent cannot be true unless the consequent is.

In saying that the truth of the consequent is necessary for the truth of the antecedent, what we are saying is that the antecedent won't be true *unless* the consequent is also true. Remember: this is **not** to say that if the consequent is true, then the antecedent will also be true! The truth of the consequent does not guarantee us the truth of the antecedent

When P is a necessary condition for S		
if	S→P	is true
and	P	is true
then	S	is not guaranteed true

Thus, true conditionals of the form "S→P" can be used to represent statements about when one condition is sufficient for another or when one condition is necessary for another.

sufficient		**necessary**
S	→	P

To conclude this section, let's return to the case where the conditional is false.

C	G	C→G
T	T	T
T	F	F
F	T	T
F	F	T

In the case of a conditional is false, the truth of the antecedent *does not* guarantee the truth of the consequent. For instance, consider the following conditional:

If a car has gas in it, then it will start.

What this says is that having gas in your car is sufficient for it starting, i.e. gas in your car guarantees it will start. This is, of course, false since cars with gas don't start for all sorts of different reasons. A false conditional shows us that the antecedent is *not* a sufficient condition for the consequent.

Before we see how this might help you with the LSAT, consider the following exercises:

Exercises Set #1:

A. Determine whether the following are *true* or *false*

1. Being a mammal is a sufficient condition for being a cat
2. Being a cat is a sufficient condition for being a mammal.
3. Being a mammal is a necessary condition for being a cat
4. Being a cat is a necessary condition for being a mammal.
5. Being arrested is a sufficient condition for being found guilty in a court of law.
6. Being arrested is a necessary condition for being found guilty in a court of law.

B. Assuming that each of the conditionals are true, identify the necessary and sufficient conditions

1. If I have ten strong alcoholic drinks, I won't be able to drive.
2. If I commit a crime, I will do the time.
3. John won't say he is sorry unless you stop talking to him.
4. Whenever I eat hot dogs, I get a stomach ache.
5. When my eye twitches, I know I've had too much caffeine.

4. Conditionals and the LSAT: An Example

Let's consider a slightly modified example problem from the "logic games" part of the LSAT (June 1994; Form 5LSS22). The logic games sections appear to consist of (i) a prompt and (ii) several questions about that prompt. The **prompt** lays down the problem and then a set of conditions that must be obeyed. Here is an example:

LSAT PROBLEM: THE PROMPT A restaurant owner is creating a work schedule for her staff: Jon, Liz, Vic, Mark, and Ryan. The schedule must meet the following conditions: • Except for Sunday & Saturday, when no one works, exactly one member of the staff works each day. • None of the staff can work more than two days per week • No staff member works on two consecutive days • Jon never works later in the week than Ryan • If Vic works, then Liz must work on the following day.

Three quick tips.

1	*Identify the goal*: note that the prompt specifies that the goal is to create a schedule for the work week
2	*Recognize the conditions*: notice that the prompt lays down conditions that *must be met* in creating a work schedule: it provides a list of conditions that are *necessary* but not *sufficient* for creating the work schedule.
3	Create a diagram: It is helpful to create an initial diagram and to use a type of shorthand.

The question prompt is followed by several different types of questions: some questions are about what can scenarios can (or cannot) happen, other questions introduce new information. Generally, these questions do not modify the initial prompt, so our initial diagram will be a useful device to answer every question about the prompt.

For the purpose of illustration, I will create the diagram in a step by step fashion. First, let's list the days and the people:

Days	M	T	W	R	F	Sa	N

People	J	L	V	M	R

Notice that we abbreviated the days of the week with a single letter, and used the first letter of each person's name to stand for that person. Next, let's begin to fill in our diagram using the necessary conditions. For the purpose of illustration, we will consider these one at a time:

Except for Sunday & Saturday, when no one works, exactly one member of the staff works each day.

Days	M	T	W	R	F	Sa	N
	x=1	x=1	x=1	x=1	x=1	OFF	OFF

None of the staff can work more than two days per week

People	J	L	V	M	R
day ≤ 2					

No staff member works on two consecutive days

Days	M	T	W	R	F
	x=1	x=1	x=1	x=1	x=1
x≠y	x	y	x	y	x

Jon never works later in the week than Ryan

People	J	L	V	M	R
day ≤ 2	before R				later than J

If Vic works, then Liz must work on the following day

People	J	V	L	M	R
day ≤ 2	before R				later than J

Consolidating our diagrams, we get the following:

People	**J**	**V**	**L**	**M**	**R**
day ≤ 2	before R				later than J

Days	**M**	**T**	**W**	**R**	**F**
	x=1	x=1	x=1	x=1	x=1
x≠y	x	y	x	y	x

With our diagram complete, we are now ready to answer some questions about the prompt:

Problem #1. Which of the following CANNOT be a Monday-to-Friday work schedule?

(A) Jon, Mark, Jon, Ryan, Mark
(B) Jon, Ryan, Vic, Liz, Jon
(C) Liz, Jon, Liz, Jon, Ryan
(D) Liz, Vic, Liz, Jon, Ryan
(E) Vic, Liz, Mark, Liz, Mark

What this question is asking is which one of these scenarios violates the necessary conditions laid down in the prompt (now represented by our diagram). To answer this question,

(1) we look at our diagram,
(2) identify a condition (or set of conditions), and then
(3) run through the different scenarios (A)–(E), until
(4) we find a violation of one of the necessary conditions.

It helps to start with the *simplest* condition to analyze and work toward the most *complex* condition to analyze. One of the easiest conditions to analyze is that no staff member can be schedule on more than two days (i.e., "day ≤ 2"). And so,

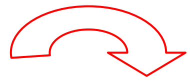
(A) Jon, Mark, Jon (2), Ryan, Mark (2)
(B) Jon, Ryan, Vic, Liz, Jon (2)
(C) Liz, Jon, Liz (2), Jon (2), Ryan
(D) Liz, Vic, Liz (2), Jon, Ryan
(E) Vic, Liz, Mark, Liz (2), Mark

Notice that (A)–(E) all pass this test and so we must move on to a more complicated condition. How about our condition that staff members cannot work on consecutive days? Represented in the diagram here:

Days	**M**	**T**	**W**	**R**	**F**
	x=1	x=1	x=1	x=1	x=1
x≠y	x	y	x	y	x

(A) Jon, Mark, Jon, Ryan, Mark
(B) Jon, Ryan, Vic, Liz, Jon
(C) Liz, Jon, Liz, Jon, Ryan
(D) Liz, Vic, Liz, Jon, Ryan
(E) Vic, Liz, Mark, Liz, Mark

Again, (A)–(E) all pass this test. How about the condition that if Vic works, then Liz must work on the following day? Represented in our diagram here:



People	J	V	L	M	R
day ≤ 2	before R				later than J

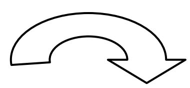
Keep in mind what it would mean to violate the following conditional:

> If Vic works, then Liz must work on the following day

A conditional “S→P” is false just in the case that the **antecedent “S” is true** and the **consequent “P” is false**. It is true in all other cases. Thus, we only need to look for a row where Vic works and Liz does *not* work on the following day:

~~(A) Jon, Mark, Jon, Ryan, Mark~~
(B) Jon, Ryan, Vic, Liz, Jon
~~(C) Liz, Jon, Liz, Jon, Ryan~~
(D) Liz, Vic, Liz, Jon, Ryan
(E) Vic, Liz, Mark, Liz, Mark

Alas, (A)–(E) still all pass this test. Let’s conclude with the final, most complex condition: *Jon never works later in the week than Ryan*, represented in our diagram as follows:



People	J	V	L	M	R
day ≤ 2	before R				later than J

We can phrase this condition in terms of the following conditional:

> If Jon and Ryan both work, then Jon never works later than Ryan.

And, from the previous example, we know that some member of (A)-(E) would make this conditional false just in the case that the antecedent of this conditional is true and the consequent is false. Similar to the previous example we only need to look for a row where Jon and Ryan both work, and then check whether Jon ever works *later than* Ryan.

(A) Jon, Mark, Jon, Ryan, Mark
(B) Jon, Ryan, Vic, Liz, Jon **– VIOLATION!**
(C) Liz, Jon, Liz, Jon, Ryan
(D) Liz, Vic, Liz, Jon, Ryan
(E) Vic, Liz, Mark, Liz, Mark

Notice that (B) is a schedule where Jon and Ryan both work *and* where Jon works later than Ryan. This violates one of the necessary conditions laid down in the prompt and so we can confidently answer this LSAT question as “B”.

Problem #2. If each staff member is required to have at least two consecutive days off during the Monday-to-Friday workweek, which one of the following could be a possible work schedule:

(A) Vic, Liz, Vic, Liz, Mark
(B) Vic, Vic, Liz, Jon, Mark
(C) Mark, Jon, Ryan, Mark, Mark
(D) Jon, Vic, Liz, Jon, Ryan
(E) Ryan, Vic, Liz, Mark, Jon

Problem #3. If both Jon and Ryan work during a week, which one of the following propositions cannot be true?
(A) Jon works on Monday and Wednesday
(B) Ryan works on Monday and Wednesday
(C) Jon works on Tuesday and Thursday
(D) Ryan works on Tuesday and Thursday
(E) Ryan works on Wednesday and Friday

Problem #4. If Jon works two days during the week and Ryan works on Thursday, which one of the following propositions could be true?
(A) Jon works on Tuesday
(B) Liz works on Tuesday
(C) Vic works on Tuesday
(D) Liz works on Wednesday
(E) Vic works on Wednesday

Problem #5. If Liz does not work during the week, which one of the following propositions must be true?
(A) Jon works exactly one day during the week.
(B) Jon works exactly two days during the week.
(C) Mark works exactly one day during the week.
(D) Mark works exactly two days during the week.
(E) Ryan works exactly one day during the week.

Adapted from February 1993, Form 3LSS18 of the LSAT

2. At a charity track & field event, an official is assigning runners–Liz, Vic, Jon, Mark, and Ryan—to five lanes, labeled 1 through 5 and to five different charities: F, G, H, J, and K. The assignments of runners to charities and to numbered lanes must meet the following restrictions:

- The runner representing K is assigned to lane 4.
- Mark is assigned to the only lane between the lanes of the runners representing F and G.
- There are exactly two lanes between Jon's lane and the lane of the runner representing G.
- Ryan is assigned a higher-numbered lane than the lane Vic is assigned.

Problem #1: Which one of the following is a possible assignment of runners to lanes by the charity they represent?

	1	**2**	**3**	**4**	**5**
(A)	F	G	H	K	J
(B)	G	H	J	K	F
(C)	G	K	F	J	H
(D)	H	J	G	K	F
(E)	J	H	F	K	G

Problem #2: The lane to which Mark is assigned must be a lane that is
(A) next to the lane to which Liz is assigned
(B) next to the lane to which Vic is assigned
(C) separated by exactly one lane from the lane to which Vic is assigned
(D) separated by exactly one lane from the lane to which Jon is assigned
(E) separated by exactly one lane from the lane to which Ryan is assigned

Problem #3: If Jon is assigned to lane 2, which one of the following assignments must be made:

	Charity	**Lane**
(A)	F	1
(B)	G	5
(C)	H	1
(D)	H	3
(E)	J	5

Problem #4: Which one of the following is a complete and accurate list of runners each of whom could be the runner representing F?

(A) Liz, Vic
(B) Mark, Ryan
(C) Liz, Vic, Jon

(D) Liz, Vic, Ryan
(E) Vic, Mark, Ryan

Problem #6: If Liz represents J, which one of the following could be the assignment of runners to lanes?

	1	**2**	**3**	**4**	**5**
(A)	Liz	Jon	Vic	Mark	Ryan
(B)	Liz	Vic	Jon	Ryan	Mark
(C)	Liz	Ryan	Mark	Vic	Ryan
(D)	Vic	Jon	Liz	Mark	Ryan
(E)	Vic	Ryan	Jon	Mark	Liz