Introduction to Symbolic Logic

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Two notions of entailment

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- A set of wffs can be said to entail a wff in two different ways

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2. A wff is **syntactically entailed** by a set of wffs iff there is a proof (or derivation) of that wff.
   - This is the notion of syntactic entailment or proof-theoretic entailment.

Derivations, entailment, the apparatus

Derivations, Syntactic Entailment, and the Deductive Apparatus

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To get a clearer sense of this **second** notion of entailment, we need to clarify a couple notions:

1. the notion of a **derivation** (or proof),
2. **deductive apparatus**: the various rules that allow for taking steps in a proof.

Once we clarify these notions, we will give a precise de nition of when a w is syntactically entailed by a set of w s.

what is a derivation (proof)?

De nition (derivation of Q in PD)

A derivation (proof) in PD of Q is a nite (not in nite and not empty) string of w s from a set of PL w s where (i) the last formula in the string is Q and (ii) each formula is either a premise, an assumption, or is the result of the preceding formulas and the deductive apparatus.

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- Every w needs to either be a premise, a w that is assumed, or a w that is proved using the deductive apparatus.

what is a deductive apparatus?

De nition (deductive apparatus)

A deductive apparatus for PL is a set of rules of inference (or \derivation" rules) that determine which ways that formulas can be transformed. That is, it is a list of permissible rules that express which wffs Q can be written after which formulas P in a proof. The deductive apparatus for PL is hereafter abbreviated as PD.

what is a deductive apparatus?

We can think of the deductive apparatus as motivated by arguing in everyday life.

- Suppose two people Tek and Liz agree on a great many things. They have similar life experiences, they read many of the same scientific studies, and they have similar values. Let's refer to the set of propositions that Tek and Liz both take as true (where just represents a set of propositions, e.g. $A; B; C; \ldots; M$).
- Now suppose that Tek thinks that from , we can easily reason to another set of propositions $P; Q; R$. Tek contends that if we believe then we ought to also believe $P; Q; R$.
- In contrast, Liz says that even if all of the propositions in are true, there is no way to reason to $P; Q; R$.

what is a deductive apparatus?

- Tek and Liz don't disagree about any facts concerning the world.
- they disagree about is whether $P; Q; R$ follows from
- To x this problem, they decide to develop a set of rules that specify how one can reason from one proposition (or groups of propositions) to another. The rules are formulated in a highly general (abstract) way so that it can apply to any particular subject matter. This set of rules is their deductive apparatus.

What , or what follows from what.

syntactic consequence

De nition (syntactic consequence (entailment))

A formula Q is a syntactic consequence in PD of a set of PL w s if and only if there is a derivation in PD of Q from .

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- P; M; R Ø Q
- What is said here is that Q is a syntactic consequence (is syntactically entailed by P; M; R
- In other words, there is a proof that begins with P; M; R and ends with Q.

How to set up a proof (Fitch-Style)

Derivation Setup (Fitch-Style)

Now that we know what a proof is, we now will look at how to set up a proof.

- Consider the following: $P, R; Y \quad R \emptyset Z$.

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First, a derivation begins with an initial setup involving three columns:

1. for numbering the premises,
2. writing (stacking) the propositions,
3. justi cation of propositions and indicating the goal proposition (or conclusion)

How to set up a proof (Fitch-Style)

1	P , R	P
2	Y R	P; Z

In the setup of the above derivation P, R and Y R are premises (and we use P) to indicate this. The conclusion (goal proposition) is Z.

Intelim Derivation Rules

The Deductive Apparatus

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What we don't have is the deductive apparatus itself !

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- Remember the deductive apparatus is just a set of rules that allow us to write new w s in the proof.

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- In other words, they are rules that allow us to go from the premises to the conclusion.

The Deductive Apparatus

- The particular type of deductive apparatus developed here is known as a system of \natural deduction" as the particular rules are akin to certain rules of inference (or reason) people use in everyday arguments.

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The Deductive Apparatus

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- In what follows, we develop the deductive apparatus $\mathcal{PD}$.
- The particular rules of $\mathcal{PD}$ will be called the \derivation rules".

The Deductive Apparatus: Intelim System

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1. introduction rules (these introduce a proposition of a certain type into a proof)
2. elimination rules (these begin from propositions of a certain type in a proof).

PD Conjunction Introduction

Conjunction Introduction (∧I)

From P and Q, we can derive (P ∧ Q). Also, from P and Q, we can derive (Q ∧ P).

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From P and Q, we can derive $P \wedge Q$. Also, from P and Q, we can derive $Q \wedge P$.

$P; Q \vdash P \wedge Q$

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The idea here is we can reason from two separate w s to their conjunction.

1. \John is a man"
2. \John is a banker"
3. Therefore, \John is a man and a banker."

PD Conjunction Introduction

1	P	P
2	Q	P
3	P , Q	, I , 1, 2

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Conjunction introduction states that from two di erent propositions, we can derive the conjunction of these propositions. For example, prove:

P Q; R - M; Z Øˆ P Q• , ˆ R - M•

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1	P Q	P
2	R - M	P
3	Z	P
4	ˆP Q• , ˆR - M•	, I , 1, 2

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Conjunction Elimination (, E)

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1 P, Z P

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1	P , Z	P
2	Z	, E, 1

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1 ˆP , Z• , Q P

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ˆP , Z• , Q ØZ

1	ˆP , Z• , Q	P
2	P , Z	, E, 1

PD Conjunction Elimination

ˆP , Z• , Q ØZ

1	ˆP , Z• , Q	P
2	P , Z	, E, 1
3	Z	, E, 2

Assumptions and Subproofs

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3. and justify that proposition you assumed with an A.

Assumptions and Subproofs

1	Main Line
2	| Assumption
3	| Subproof
4	| :
5	| :
6	| :

Assumptions and Subproofs

Some quick notes about assumptions

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- Making an assumption and reasoning under that assumption is something everyone does.
  1. Suppose you and a friend are having an argument.
  2. You might say to your friend, \let's assume what you are saying is true" and then reason based on that assumption.

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After you have made an assumption, you can reason within the subproof that has been created by the assumption:

1	S		P
2	|	B	A
3	|	S , B	, I , 1, 2
4	|	:	
5	|	:	
6	|	:	

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After you have made an assumption, you can reason within the subproof that has been created by the assumption:

1	S		P
2	|	B	A
3	|	S , B	, I , 1, 2
4	|	:	
5	|	:	
6	|	:	

The above is similar to saying Lets agree that S is true. Now, lets assume B is true. Well, if S is true, then given our assumption B, it follows that S, B

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1	Q			P
2	|	S		A
3	|	|	W	A
4	|	|	:	
5	|	|	:	
6	|	|	:	

The prior proof proof reads something like the following: lets say that Q is true. Given that Q is true, lets assume S. Now that weve assumed S, lets assume W.

Assumptions and Subproofs

You can think of subproofs like containers or nests. That is the subproof begun by S contains the subproof begun by W. Likewise, the mainline of the proof, beginning with Q contains the subproofs begun by S and W. In the language of nests, W is in the nest begun by S and S is in the nest of the main line of the proof. W is in the most deeply nested part of the proof while Q is in the least deeply nested part.

Assumptions and subproofs can be independent of each other.

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1	A		P
2		B	A
3		A , B	, I , 1, 2
4		:	
5		:	
6		C	A
7		A , C	, I , 1, 6
8		:	
9		:	

Assumptions & Subproofs

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- know what an assumption is and that assumptions start subproofs
- know that subproofs can be nested or independent of each other
- know that you can reason within a subproof (under an assumption)
- examine some general rules for reasoning in a subproofs and some violations of those rules

Assumptions & Subproofs

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Deriving w s from a subproof

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Deriving w s from a subproof

In other words, If R is in a section of the proof S_3 that does not contain a subproof S_4, then R cannot be used in S_4.

Assumptions & Subproofs: Violations!

The above rule is violated when using a proposition inside a subproof, you derive a proposition outside the subproof.

Assumptions & Subproofs: Violations!

Example # 1: Z is in the subproof and used to derive Z, R, a proposition which is not in the subproof containing Z

1	R	P
2	| Z	A
3	| R, Z	, I, 1, 2
4	Z, R; NO!	, I, 1, 2

Assumptions & Subproofs: Violations!

Example # 2: B is in the subproof and used to derive B, C, a proposition which is not in the subproof containing B

1	A		P
2		B	A
3		A, B	, I, 1, 2
4		C	A
5		B, C; NO!	, I, 2, 4

Reasoning with Subproofs

- know what an assumption is and general rules for reasoning in a subproofs and some violations of those rules

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Reasoning with Subproofs

- know what an assumption is and general rules for reasoning in a subproofs and some violations of those rules
- but what can we do with subproofs?
- speci c derivation rules that allow you to reason from a subproof to a w in the proof.

Conditional Introduction (I)

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Conditional introduction says that from a subproof that begins with the assumption P and ends with Q, we can derive the conditional P Q.

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n		P	A
ˆn 1•		Q	
ˆn 2•	P Q		I, n{ˆn 1•

Conditional introduction

Here is an example: $R \varnothing Z \quad R$

Conditional introduction

Here is an example: R Ø Z R

1	R		P
2	|	Z	A
3	|	Z , R	, I , 1, 2
4	|	R	, E, 3
5	Z	R	I , 2{4

Conditional introduction

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- Let's consider an example in plain English:
  1. Assume that a nuclear bomb is dropped on my house (Note: I am not saying that one has been dropped on my house or will be dropped on my house).
  2. Under this assumption, I can infer that my house will be destroyed.
  3. Therefore, I can reason using conditional introduction to the complex proposition: if a nuclear bomb is dropped on my house, then it will be destroyed.

Conditional introduction

Conditional Elimination (E)

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From $P \quad Q$ and P, we can derive Q.

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$P \quad Q; P \varnothing Q$

Conditional Elimination (E)

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From P Q and P, we can derive Q.

P Q; P Ø Q

Conditional elimination allows for deriving a proposition Q provided we have a conditional P Q and the antecedent P of that conditional.

Conditional Elimination (E)

Here is an example: Z R; Z , P Ø R

Conditional Elimination (E)

Here is an example Z R; Z , P Ø R

1	Z R	P
2	Z , P	P , R
3	Z	, E, 2
4	R	E, 1, 3

Conditional Elimination (E)

Reiteration

Reiteration (R)

From P we can derive P.

Reiteration

Reiteration (R)

From P we can derive P.

P ØP

Reiteration

Reiteration (R)

From P we can derive P.

P ØP

Reiteration allows for deriving a proposition P provided P already occurs in the proof.

Reiteration

Here are two examples.

Reiteration

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Example # 1: Z ØZ

Reiteration

Here are two examples.

Example # 1: Z ØZ

1	Z	P; Z
2	Z	R, 1

Reiteration

Example # 2: R ØZ R

Reiteration

Example # 2: R ØZ R

1	R		P; Z R
2		Z	A
3		R	R, 1
4	Z R		I , 2{3

Negation Introduction

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From a derivation of a proposition Q and its literal negation ¬Q within a subproof involving an assumption P, we can derive ¬P out of the subproof.

Negation Introduction

Negation Introduction (I)

From a derivation of a proposition Q and its literal negation Q within a subproof involving an assumption P, we can derive P out of the subproof.

n	P	A
ˆn 1•	Q	
ˆn 2•	Q	
ˆn 3•	ˆP•	I, n{ˆn 2•

Negation Elimination

Negation Elimination (E)

From a derivation of a proposition Q and its literal negation Q within a subproof involving an assumption ˆP•, we can derive P out of the subproof.

n	ˆP•	A
ˆn 1•	Q	
ˆn 2•	Q	
ˆn 3• P		E, n{ˆn 2•

Negation Elimination

Example : P , P Ø Z R

Negation Elimination

Example : P , P ØZ R

1	P ,	P	P ; Z R
2	|	ˆZ R•	A
3	|	P	, E, 1
4	|	P	, E, 1
5	Z	R	E, 2{4

Negation Elimination and Introduction

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Negation Elimination and Introduction

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2. sometimes these proofs are called proof by contradiction
3. both are a form of indirect proof as they prove that something is the case by (i) assuming the opposite proposition and (ii) showing that proposition to lead to contradiction (Q; Q).

Negation Elimination and Introduction

1. E and I are a species of reductio ad absurdum (\reduction to absurdity")
2. sometimes these proofs are called *proof by contradiction*
3. both are a form of *indirect proof* as they prove that something is the case by (i) assuming the *opposite* proposition and (ii) showing that proposition to lead to contradiction (Q; Q).
4. How might you use E and I in everyday life?

Negation Elimination and Introduction

1. Premises: If God exists, then God is all-loving, all-knowing, and all-powerful. If God is all-loving, all-knowing, and all-powerful, then there should be no evil in the world. There is evil in the world.

Negation Elimination and Introduction

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Negation Elimination and Introduction

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Negation Elimination and Introduction

1. Premises: If God exists, then God is all-loving, all-knowing, and all-powerful. If God is all-loving, all-knowing, and all-powerful, then there should be no evil in the world. There is evil in the world.
2. Suppose you wanted to accept the above and you want to prove that God does not exist.
3. Start by assuming the opposite of what you want to prove. That is, you would argue by saying: Let's assume that God does exist.
4. Then you would try to show that under this assumption, a contradiction follows (there should be no evil in the world and there is evil in the world), and so ultimately conclude that God does not exist.

Negation Elimination and Introduction: FAQs

Time for some frequently asked questions!

Negation Elimination and Introduction: FAQs

Question

What should I assume?

Negation Elimination and Introduction: FAQs

Question

What should I assume?

You will always assume the opposite (the negation) of the w you want.

Negation Elimination and Introduction: FAQs

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1. If you want to derive P (it's your conclusion), then assume P

Negation Elimination and Introduction: FAQs

Question

What should I assume?

You will always assume the opposite (the negation) of the w you want.

1. If you want to derive P (it's your conclusion), then assume $\neg P$
2. If you want to derive $P \wedge Q$, then assume $\neg(P \wedge Q)$
3. If you want to derive Z, then assume either $\neg Z$ or Z

Negation Elimination and Introduction: FAQs

Question

When is it a good time to use E and I?

Negation Elimination and Introduction: FAQs

Question

When is it a good time to use E and I?

1. when the premises are inconsistent.

Negation Elimination and Introduction: FAQs

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   - Derive and ˆ •, then assume the opposite of the conclusion, reiterate and ˆ • into the subproof, then use either E and I

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1. when the premises are inconsistent.
   - Derive and ˆ •, then assume the opposite of the conclusion, reiterate and ˆ • into the subproof, then use either E and I
2. when you can't use any more elimination rules to simplify your proof or you don't know what to do.

Negation Elimination and Introduction: FAQs

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When is it a good time to use E and I?

1. when the premises are inconsistent.
   - Derive and ˆ •, then assume the opposite of the conclusion, reiterate and ˆ • into the subproof, then use either E and I
2. when you can't use any more elimination rules to simplify your proof or you don't know what to do.
   - Assume the opposite of the conclusion, then see if this assumption will allow you to derive and ˆ •. If it does, then you can use either E and I to solve.

Negation Elimination and Introduction: FAQs

Question

When is it NOT a good time to use E or I?

Negation Elimination and Introduction: FAQs

Question

When is it NOT a good time to use E or I?

1. when you don't have a clear reason to do it and you can use elimination rules (e.g., , E; E) to simplify your proof further

Negation Elimination and Introduction: FAQs

Question

When is it NOT a good time to use E or I?

1. when you don't have a clear reason to do it and you can use elimination rules (e.g., , E; E) to simplify your proof further
2. Maybe: when your conclusion is a conditional. Here it might make more sense to try and use I

Negation Elimination and Introduction: FAQs

Question

When can I use these rules in daily life?

Negation Elimination and Introduction: FAQs

Question

When can I use these rules in daily life?

1. practically: hard to say. You are showing a position to be absurd.

Negation Elimination and Introduction: FAQs

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1. practically: hard to say. You are showing a position to be absurd.
2. argumentative situation: when you think your opponent's position entails an absurdity (you can assume their position to be true, draw out the contradiction, and then reason to the conclusion that their position is false)

Negation Elimination and Introduction: FAQs

Question

When can I use these rules in daily life?

1. practically: hard to say. You are showing a position to be absurd.
2. argumentative situation: when you think your opponent's position entails an absurdity (you can assume their position to be true, draw out the contradiction, and then reason to the conclusion that their position is false)
3. personal situation: use as a way of self-critique. Suppose I think that (i) character is determined by what you do on a regular basis, (ii) I don't give to charity (or at all). I might assume I'm charitable and reason to a contradiction.

Negation Elimination and Introduction: FAQs

Question

Can I see you use this rule one more time? Example please!

Negation Elimination and Introduction: FAQs

Disjunction Introduction

Disjunction Introduction (- I)

In the case of disjunction introduction-(I), we reason as follows:

- P1: Proposition P is the case.
- C: Therefore, P or Q is the case.

Disjunction Introduction

People tend not to reason using $\vee$ in everyday life. But, if they did, it would be perfectly acceptable.

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Example

1. P1: I will de nitely take another philosophy class next semester.

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1. P1: I will definitely take another philosophy class next semester.
2. C: Therefore, I will definitely take another philosophy class next semester OR a biology class.

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1. P1: I will definitely take another philosophy class next semester.
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Example

1. P1: John will look at the stars tonight.

Disjunction Introduction

People tend not to reason using $\vee I$ in everyday life. But, if they did, it would be perfectly acceptable.

Example

1. P1: I will definitely take another philosophy class next semester.
2. C: Therefore, I will definitely take another philosophy class next semester OR a biology class.

Example

1. P1: John will look at the stars tonight.
2. C: Therefore, John will look at the stars tonight OR binge watch Netflix.

Disjunction Introduction (- I)

Disjunction introduction (as the name implies) is a derivation rule that introduces into the proof a disjunction (an \or" proposition, a w where is the main operator.

Disjunction Introduction

Disjunction Introduction (- I)

From a w , - or - can be derived.

- Ø -
- Ø -

Disjunction Introduction

- Sometimes it isn't obvious why $\vee$I is a legitimate derivation rule
- Its acceptance can be seen through a truth-table analysis of the derivation

P Ø P - Q

P	Q	P	Ø	P - Q
T	T	T		T
T	F	T		T
F	T	F		T
F	F	F		F

Notice that whenever P is true, P - Q is true. We can never go from a true premise to a false conclusion!

Double Negation (DN)

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Where P is any PL-w , from P, we can derive ˆP• and from ˆP•, we can derive P.

P ÚØ ˆP•

Double Negation (DN)

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P ÚØ ˆP•

DN allows for replacing a single formula or single subformula with its doubly negated form or taking a doubly negated formula and replacing it with its unnegated form. For example,

Double Negation (DN)

1	P R	P, ˆP R•
2	ˆP R•	DN, 1

Double Negation (DN)

It is important to note that replacement rules can be applied to a single subformula. For example,

1	P - ˆR , S•	P¬P - ˆR , S•
2	P - ˆR , S•	DN, 1

Double Negation (DN)

B

ut, be careful! DN must be applied to the whole of a formula or subformula and not to part of one subformula and part of another subformula:

1	P - ˆR , S•	P
2	P - ˆ R , S•; NO!	DN, 1

Disjunction Introduction

Tricky Point

One strange thing about ∨I is that you can reason from a w to a disjunction - and the w can be any w (even a contradiction!). This is because if v^ • T , then v^ - • T .

Disjunction Introduction

Tricky Point

One strange thing about ∨I is that you can reason from a w to a disjunction - and the w can be any w (even a contradiction!). This is because if v^ • T , then v^ - • T .

Example

- P1: P
- C: P - ^Q - S•

If P is true, then P - ^Q - S• is true since a disjunction is true provided one of its disjuncts are true.

Disjunction Introduction

Let's look at some examples of in a proof. Let's prove R ØR - Q

Disjunction Introduction

Let's look at some examples of I in a proof. Let's prove R Ø R - Q

1	R	P
2	R - Q	- I , 1

Notice that the justi cation for line (2) is line (1) and we cite I to let everyone know what derivation rule we used.

Disjunction Introduction

Let's look at some examples of in a proof. Let's prove P Ø ˆ P - Q • - R

Disjunction Introduction

Let's look at some examples of in a proof. Let's prove P Ø ˆP - Q• - R

1	P	P
2	P - Q	- I, 1
3	ˆP - Q• - R	- I, 2

Disjunction Introduction

Generally, there is no reason to use unless you know you want a disjunction . Using this rule to use it just makes the proof more complicated.

Suppose our proof is P - Q • R; P Ø R

Disjunction Introduction

Generally, there is no reason to use unless you know you want a disjunction . Using this rule to use it just makes the proof more complicated.

Suppose our proof is P - Q• R; P Ø R

1	ˆP - Q•	R	P
2	P		P

Question: What rule should we use?

Disjunction Introduction

Prove: ˆP - Q• R; P ØR

1	ˆP - Q• R	P
2	P	P

One idea is that if we had the disjunction P - Q, then we could use one of our other rules (e.g. E ; Ø) to derive R.

Disjunction Introduction

Prove: ˆP - Q• R; P ØR

1	ˆP - Q•	R	P
2	P		P
3	P - Q		- I, 2

Disjunction Introduction

Prove: ˆP - Q• R; P ØR. Now that we have P - Q, we can use conditional elimination to derive our conclusion.

1	ˆP - Q• R	P
2	P	P
3	P - Q	- I, 2
4	R	E, 1, 3

Disjunction Elimination

Disjunction Elimination

Disjunction elimination is perhaps the most complicated rule. Here is the basic idea:

- Start from a disjunction (Either P or Q).
- Assume P is the case and show that R follows from P.
- Now assume Q is the case and show that R follows from Q.
- Since you know either P or Q is the case, and you know that R follows from both P and Q, you can conclude that R is the case.

Disjunction Elimination

First, Let me give you a visual example of this rule. Text of this visual example follows in case you forget what these crazy pictures mean.

Disjunction Elimination

- I am either going to stay home or I am going to party.

Disjunction Elimination

- I am either going to stay home or I am going to party.
- Will I have a good time tonight?

Disjunction Elimination

- I am either going to stay home or I am going to party.
- Will I have a good time tonight?
- Well, assume I stay home, and I know that if I did this, I would have a good time.

Disjunction Elimination

- I am either going to stay home or I am going to party.
- Will I have a good time tonight?
- Well, assume I stay home, and I know that if I did this, I would have a good time.
- Will I have a good time tonight? If I stay home, I will, but I might decide to party instead. So we need to see what happens if I party!

Disjunction Elimination

- I am either going to stay home or I am going to party.
- Will I have a good time tonight?
- Well, assume I stay home, and I know that if I did this, I would have a good time.
- Will I have a good time tonight? If I stay home, I will, but I might decide to party instead. So we need to see what happens if I party!
- Now assume that I party all night long, and I know that if I did this, I would have a good time.

Disjunction Elimination

- I am either going to stay home or I am going to party.
- Will I have a good time tonight?
- Well, assume I stay home, and I know that if I did this, I would have a good time.
- Will I have a good time tonight? If I stay home, I will, but I might decide to party instead. So we need to see what happens if I party!
- Now assume that I party all night long, and I know that if I did this, I would have a good time.
- Will I have a good time tonight? If I party all night long, then I will have a good time.

Disjunction Elimination

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- Now assume that I party all night long, and I know that if I did this, I would have a good time.
- Will I have a good time tonight? If I party all night long, then I will have a good time.
- RESULT: Since we know I'm either going to stay home or party, and we know that no matter what I do, I will have a good time, we can conclude I will have a good time.

Disjunction Elimination

NOTE!

Keep in mind that we are only saying \I will have a good time" follows from the premises. Not that \I will have a good time" is true. Logic is about drawing correct inferences from the premises! It might be the case that I have a terrible time since the disjunction is false: I might decide to neither stay home nor party.

Disjunction Elimination

Back to symbolic logic. The disjunction elimination rule ($\vee E$) states that from a disjunction - and two derivations of a w (where can be any w), the rst where is assumed and is derived, the other where is assumed and is derived, we can derive .

Disjunction Elimination

Disjunction Elimination (- E)

1	P - Q	P
n	P	A
ˆn 1•	R	
ˆi•	Q	A
ˆi 1•	R	
ˆk•	R	- E, 1, n{ˆn 1•, ˆi•{ˆi 1•

Disjunction Elimination

Let's look at an example where we try to prove the following:

P - Q; P R; Q R Ø R

Disjunction Elimination

Prove: P - Q; P R; Q R ØR. First, set up the proof.

1	P - Q		P
2	P	R	P
3	Q	R	P

Disjunction Elimination

Prove: P - Q; P R; Q R Ø R. Next, let's assume P as it is the left disjunct. In doing this, we want to derive a w . Let's try for R since it is our conclusion!

1	P - Q	P	
2	P R	P	
3	Q R	P	
4	| P	A	
5	| R		E, 2, 4

Disjunction Elimination

Prove: P - Q; P R; Q R Ø R. Now, let's assume P as it is the right disjunct. In doing this, we want to derive the same w as before, namely R.

1	P - Q	P
2	P R	P
3	Q R	P
4	| P	A
5	| R	E, 2, 4
6	| Q	A
7	| R	E, 3, 6

Disjunction Elimination

Prove: P - Q; P R; Q R ØR. Finally, use - E to derive R. Cite the rule (- •, the disjunction (line 1), the rst subproof (lines 4-5) and the second subproof (lines 6-7).

1	P - Q	P
2	P R	P
3	Q R	P
4	| P	A
5	| R	E, 2, 4
6	| Q	A
7	| R	E, 3, 6
8	R	- E, 1, 4{5, 6{7

Disjunction Elimination

My general advice about $\vee$E is to use it (1) as a last resort and (2) only when you have a disjunction already in the proof.

Biconditional Introduction

Biconditional Introduction (I)

Biconditional introduction , as the name implies, is a derivation rule that introduces into the proof a biconditional.

Biconditional Introduction (I)

Suppose I can prove two propositions:

- If taxes go up, then there will be a recession.
- If there is a recession, then taxes have gone up.

Biconditional introduction is the procedure of proving the two propositions above and then derives the following biconditional:

- Taxes go up if and only if there will be a recession.

Biconditional Introduction (I)

It does this in the following way:

1. First, assume , then show that follows.
2. Second, assume, then show that follows.
3. Finally, derive from (1) and (2).

Biconditional Introduction

Biconditional Introduction (I)

n	P	A
ˆn 1•	Q	
ˆi•	Q	A
ˆi 1•	P	
ˆk• P Q	I, n{ˆn 1•, ˆi•{ˆi 1•	

Biconditional Introduction

Let's consider an example where we prove: $P \leftrightarrow Q$, $P \rightarrow Q$

Biconditional Introduction

Prove: $P; Q \varnothing P \quad Q$. First, set up the proof.

1	P	P
2	Q	P

Biconditional Introduction

Prove: $P; Q \varnothing P \quad Q$. Next, we need to assume P, then show Q follows.

1	P	P
2	Q	P
3	| P	A
4	| Q	R, 2

Biconditional Introduction

Prove: $P; Q \varnothing P$ Q. Now, we need to assume Q, then show P follows.

1	P		P
2	Q		P
3		P	A
4		Q	R, 2
5		Q	A
6		P	R, 1

Biconditional Introduction (I)

Prove: P; Q ØP Q. Finally, we can derive P Q using

1	P		P
2	Q		P
3		P	A
4		Q	R, 2
5		Q	A
6		P	R, 1
7	P Q		I, 3{4, 5{6

Biconditional elimination (E)

Biconditional Elimination (E)

In the case of biconditional elimination (E), we reason as follows:

Biconditional Elimination (E)

In the case of biconditional elimination (E), we reason as follows:

- P1: P if and only if Q
- P2: P.
- C: Therefore, Q .

OR

- P1: P if and only if Q
- P2: Q.
- C: Therefore, P.

Biconditional Elimination (E)

Some examples:

Biconditional Elimination (E)

Some examples:

Example

1. P1: John is a bachelor if and only if he is not married.

Biconditional Elimination (E)

Some examples:

Example

1. P1: John is a bachelor if and only if he is not married.
2. P2: John is not married.

Biconditional Elimination (E)

Some examples:

Example

1. P1: John is a bachelor if and only if he is not married.
2. P2: John is not married.
3. C: Therefore, John is a bachelor.

Example

1. P1: I will go to heaven if and only if I am forgiven.

Biconditional Elimination (E)

Some examples:

Example

1. P1: John is a bachelor if and only if he is not married.
2. P2: John is not married.
3. C: Therefore, John is a bachelor.

Example

1. P1: I will go to heaven if and only if I am forgiven.
2. P2: I will go to heaven.

Biconditional Elimination (E)

Some examples:

Example

1. P1: John is a bachelor if and only if he is not married.
2. P2: John is not married.
3. C: Therefore, John is a bachelor.

Example

1. P1: I will go to heaven if and only if I am forgiven.
2. P2: I will go to heaven.
3. C: Therefore, I am forgiven.

Biconditional Elimination (E)

Biconditional Elimination (E)

From P Q and P, we can derive Q. And, from P Q and Q, we can derive P.

P Q; P Ø Q

P Q; Q Ø P

Biconditional Elimination (E)

1	P	Q	P	
2	P		P	
3	Q			E, 1, 2

Biconditional Elimination (E)

Let's solve a proof using E. Prove: ˆP , Q• S; P; Q ØS.

Biconditional Elimination (E)

Let's solve a proof using E. Prove: ˆP , Q• S; P; Q ØS. First, set up the proof correctly.

1	ˆP , Q• S	P
2	P	P
3	Q	P

Question: Can we solve this proof in one step using E?

Biconditional Elimination (E)

Prove: ˆP , Q• S; P; Q ØS. Second, we see there is a biconditional at line 1. We need one side of the biconditional to reason to the other side. Let's derive the left side P , Q.

1	ˆP , Q• S	P
2	P	P
3	Q	P
4	P , Q	, I , 2, 3

Biconditional Elimination (E)

Prove: ˆP , Q• S; P; Q ØS. Now, we have the left side of the biconditional. We can use E to derive the right side (which is our conclusion!)

1	ˆP , Q• S	P
2	P	P
3	Q	P
4	P , Q	, I , 2, 3
5	S	E, 1, 4

Proofs: Strategies

Frame Title

There are two main types of strategies: proof strategies and assumption strategies.

SP# 1 (E)	First, eliminate any conjunctions with E, disjunctions with - E, conditionals with E, and biconditionals with E. Then, if necessary, use any necessary introduction rules to reach the desired conclusion.
SP# 2 (B)	First, work backward from the conclusion using introduction rules (e.g., I; - I; I; I). Then, use SP# 1 (E).

Table 1: Proof Strategies

SA# 1 (P; Q)	If the conclusion is an atomic w (or a negated w), assume the negation of the w (or the non-negated form of the negated w), derive a contradiction and then use I or E.
SA# 2 ( )	If the conclusion is a conditional, assume the antecedent, derive the consequent, and use I.
SA# 3 (,)	If the conclusion is a conjunction, you will need two steps. First, assume the negation of one of the conjuncts, derive a contradiction, and then use I or E. Second, in a separate subproof, assume the negation of the other conjunct, derive a contradiction, and then use I or E. From this point, a use of , I will solve the proof.
SA#4 (-)	If the conclusion is a disjunction, assume the negation of the whole disjunction, derive a contradiction, and then use or E.

Frame Title

Consider ˆ P , Q• ØP - Q. The strategy associated with assumptions is SA#3.

1	ˆ P , Q•	P; P - Q
2	| ˆP - Q•	A¬P; P
	|	

Frame Title

The subgoal at this point is to generate a proposition P and its literal negation in the subproof, but it is not clear how to do this. You cannot generate P and P out of nothing so consider what propositions you do have and try to derive a proposition that is a literal negation of these.

1	ˆ P , Q•	P; P - Q
2	| ˆP - Q•	A~P; P
#	| P , Q or P - Q	?

Frame Title

We thus have two options:

Option # 1: derive P , Q since ˆ P , Q• is its literal negation Option # 2: deriveP - Q since ˆP - Q• is its literal negation

Frame Title

Consider option # 2. If we were to try to derive P - Q, we need to make an assumption, and the strategic rule associated with deriving disjunctions SA#4 says to assume the negation, derive P and P, and then use E or I. In the case of the above proof, the next step would be as follows:

1	ˆ P , Q•	P; P - Q
2	| ˆP - Q•	A¬P; P
3	| | ˆP - Q•	A¬P; P
#	| | P , Q or P - Q	?

Frame Title

But this does not help since we still have no way to get P in the proof. So, consider option # 1. If we were to try and derive $P \wedge Q$, we would need to make an assumption, and the strategic rule associated with conjunctions SA#3 says to assume the literal negation of each of the conjuncts in separate subproofs, derive P and $\neg P$ in each, and then use $\neg I$ or $\neg E$.

Frame Title

1	ˆ P , Q•	P ; P - Q
2	| ˆP - Q•	A¬P; P
3	| | P	A¬P; P
	| |	
n	| | Q	A¬P; P
	| |	

Frame Title

Now the proof can be more easily solved.

Proofs: Additional Derivation Rules (PD+)

Frame Title

The set of 10 intelim rules along with reiteration forms PD, a derivation system capable of proving any valid argument in PL. In other words, PD consists of all of the essential derivation rules we need. However, you may have noticed that the proofs for many straightforwardly valid arguments are overly di cult or time-consuming. For example, the proof of P - Q; Q Ø P is overly complicated given that the argument is straightforwardly valid. In what follows, a number of additional derivation rules are added to PD to form PD+. These additional derivation rules serve to expedite the proof solving process.

Disjunctive Syllogism

Disjunctive Syllogism (DS)

Disjunctive syllogism (DS) is a derivation rule in propositional logic.

1. it is a derived rule in an intelim system
2. made use of because (i) it corresponds to how people reason naturally and (ii) simpli es certain proofs

Disjunctive Syllogism (DS)

Disjunctive Syllogism (DS)

From - and ˆ •, we can derive .

From - and ˆ •, we can derive .

- ; ˆ • Ø
- ; ˆ • Ø

Disjunctive Syllogism (DS): Simple Example

1	P - R	P
2	R	P
3	P	DS, 1, 2

Disjunctive Syllogism (DS): English Example

- P1: There is cake or ice cream. $C \vee I$
- P2: It is not the case that there is ice cream. $\neg I$.
- C: Therefore, there is cake. C

Disjunctive Syllogism (DS): Justi cation

The rule is justi ed in two di erent ways:

1. since DS is derived, we can prove ; ˆ • Ø using a more basic set of derivation rules.
2. intuitive justi cation from cases

Disjunctive Syllogism (DS): Intuitive justi cation

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- P2: It is not the case that there is ice cream. (I).
- C: Therefore, there is cake.Ɑ•

- P1: (1) cake, (2) ice cream, (3) cake and ice cream

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Disjunctive Syllogism (DS): Example

1	P - ˆM T •	P
2	ˆM T •	P
3	P	DS, 1, 2

Modus Tollens (MT)

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From $P \to Q$ and $\neg Q$, we can derive $\neg P$.

$P \to Q;\ \neg Q \vdash \neg P$

The general idea is that given a conditional $P \to Q$ and the literal negation of the consequent $\neg Q$, the negation of the antecedent $\neg P$ can be derived.

Modus Tollens (MT)

1	P ˆS - R•	P
2	ˆS - R•	P
3	P	MT, 1, 2

Hypothetical Syllogism

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From P Q and Q R, we can derive P R.

Hypothetical Syllogism (HS)	
1	P
2	P
3	HS, 1, 2

The idea is that if you have two conditionals P Q and Q R where the consequent of one conditional P Q is the antecedent of the other conditional Q R, then you can derive a third conditional P R.

Hypothetical Syllogism (HS)

In HS, we are reasoning from two conditionals to a third conditional.

1	P
2	P
3	HS, 1, 2

- notice that the consequent of one conditional is the antecedent of another conditional.
- notice that the conditional we reason to is the antecedent of one conditional and the consequent of another

Hypothetical Syllogism (HS)

Individuals use HS in everyday reasoning all the time. Here is an example:

- P1: If I go to the store, I'll buy some cigarettes.
- P2: If I buy cigarettes, I'll end up smoking.
- C: Therefore, if I go to the store, I'll end up smoking.

Notice you have the same proposition in part of each conditional!