Handout 1

ELEMENTS OF LOGIC

1.1 What is Logic? Arguments and Propositions

In our day to day lives, we find ourselves *arguing* with other people. Sometimes we want someone to do or accept something as true and so we find ourselves citing various reasons for why they ought to do what we want or accept what we think to be the case. Other people are no different. They wish to convince us of something and so they use various tactics to get our assent. Sometimes we find their tactics convincing, other times we are unpersuaded, and other times we are unsure what we should think.

Logic is a science that aims to identify principles for good and bad reasoning. In essence, it aims to develop criteria (rules or methods) for the identification of good and bad arguments.

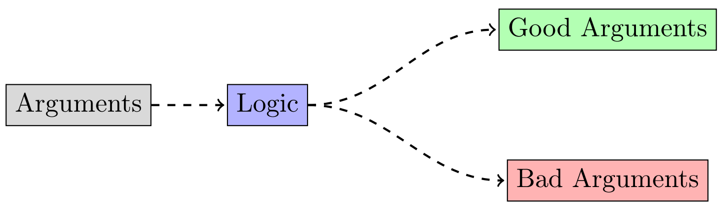

Figure 1.1 – Logic aims to identify good arguments

In formulating criteria for good and bad arguments, the focus of logic is on *arguments*, not on *arguing*. Arguing and arguments differ in many important ways. Arguing is a broader, more complex phenomena that often, although not always, includes arguments. When we argue, it is often directed at some other person (our friends, family, or strangers), it can involve yelling, the rolling of our eyes in disbelief, or some other gesture. In contrast, an "argument" is a sequence of sentences (called "propositions") where some proposition (called the "conclusion") is represented as following from another set of propositions (called the "premises").

Definition – Argument

An argument is a series of propositions in which a certain proposition (the conclusion) is represented as following from another set of propositions (the premises or assumptions).

One way to think about the concept of an argument is that an argument is an *abstraction* (or selection) from what goes on when people argue. That is, part of what people do (or at least aim to do) when they argue is to utter sentences that support some other sentence. In doing this, they put forward arguments.

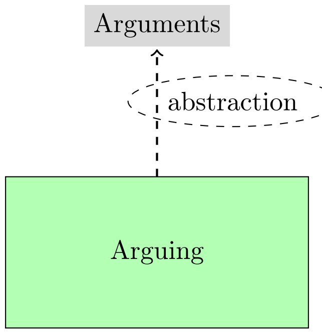

Figure 1.2 – An argument can be understood as an abstraction from the everyday practice of arguing

1.1.1 Propositions

The definition of an argument contains a number of terms that need clarification. First, an argument is a series of propositions. What is a proposition?

Definition – Proposition

A proposition is a sentence (or something that is expressed by a sentence) that is capable of being true or false.

Example 1: Examples of Propositions

1. John is tall.
2. There are 10,234 trees in State College.
3. Socrates is Mortal.

Many types of things do not express propositions. For example:

Questions: Is John tall?
Exclamations: Woah!
Commands: Close the door.
Performative Utterances: ‘I do’ uttered on your wedding day.

Key Ideas about Propositions

1. Sentence $\neq$ Proposition: While all propositions are expressed by sentences, not all sentences express propositions, e.g. commands, questions, exclamations do not express propositions.

2. Many Sentences Can Express One Proposition: A single proposition can be expressed in a variety of different ways

   (a) Example 1: 'John loves Liz' vs. 'Liz is loved by John.'
   (b) Example 2: A single proposition expressed in two different languages

3. One Sentence Can Express Many Propositions: A single sentence does not always express the same proposition, e.g. 'I ate breakfast'

4. Being T or F vs. Knowing T of F: While a proposition must express content that is true or false (or can be true or false), it is not necessary that you know the truth value of a sentence (or know how to confirm the truth value) in order for the sentence to be a proposition, e.g. 'there are 50,304 trees in State College.'

Exercise 1: *Determine whether the following sentences express propositions.*

1. Be a yardstick of quality.
2. Don't make friends with drug dealers.
3. How may I help you?
4. I know that God exists.
5. God does not exist.

1.1.2 Premises and the Conclusion

Two other key terms in the definition of an argument are "premises" and "conclusion". The terms "premises" and "conclusion" refer to the role certain propositions play in an argument. Intuitively, the "premises" or the "premise" of an argument plays a supporting role in that they serve to "support" or act as "evidence" or reasons for a conclusion. The "conclusion" plays the role of proposition that is supported.

To illustrate, note that in Argument 1.1 there are three propositions, labeled (P1), (P2), and (C). (P1) and (P2) are the premises and they support (C), the conclusion.

1.1.3 How to Identify Arguments

An argument is not a mere list of sentences nor is it a list of propositions. For example, some sets of sentences are not wholly composed of propositions (e.g., a list of questions) and so they are not arguments. In other cases, there are a

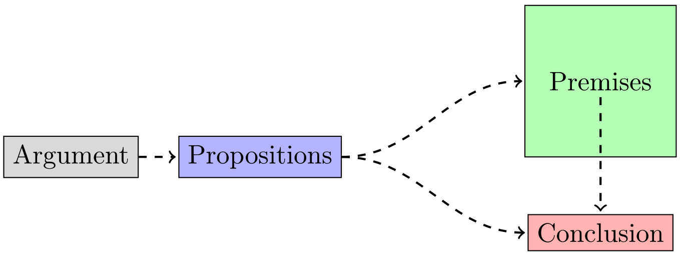

Figure 1.3 – Arguments consists of propositions (premises and a conclusion)

P1	All women are mortal.	Premise
P2	Liz is a woman.	Premise
C	Therefore, Liz is mortal.	Conclusion

Argument 1.1 – An argument consisting of three propositions, two premises, and a single conclusion.

series of sentences, all of which propositions, but these propositions don't take on the role of "premise" or "conclusion" (e.g. a descriptive account of your day is not an argument). Identifying what is and what is not an argument is an art.

1. Look to see if the sentences or words consist of propositions.
2. Identify certain key terms called "argument indicators" like 'therefore,' 'in conclusion,' 'since,' 'it follows that' that indicate the presence of an argument.
3. Determine whether one proposition (the conclusion) can plausibly be said to logically follow from another set of propositions (the premises or an assumption). For example, suppose three propositions P1, P2, P3, test whether they are an argument by writing P1, P2, therefore P3, then P1, P3, therefore P2.
4. Look for certain conventional practices in how arguments are expressed, e.g. its presentation in a schematic or standard form.

Indicators of Premises	Indicators of Conclusions
because ... , for, since, for the reason that ... , is supported by ..., may (or can) be deduced from, ... is a reason, ... suggests	therefore, hence, in conclusion, so, entails, implies that, indicates, therefore, consequently, we may (can) deduce that, suggests ..., it follows that ..,

Table 1.1 – Common Argument Indicators

While arguments do not have a single order of presentation, a standard way of presenting arguments is as follows:

First	Premises/Assumptions
Second	Argument Indicator
Third	Conclusion

Table 1.2 – A Diagrammatic Presentation of an Argument

Example 1:

John told me that David is a bad teacher. Frank also told me that David is a bad teacher. John and Frank are never wrong about who is a good or bad teacher because they have failing grades. Therefore, David must be a bad teacher.

P1	John told me that David is a bad teacher.	Premise
P2	Frank also told me that David is a bad teacher.	Premise
P3	John and Frank are never wrong about who is a good or bad teacher because they have failing grades.	Premise
C	Therefore, David must be a bad teacher.	Conclusion

Argument 1.2 – argument indicator 'therefore'

Exercise 2: *Classroom Exercise: Create Your Own Argument!*

In a small group, create your own argument. Make sure you label the premises, the conclusion, and any terms that be considered argument indicators.

1.2 Evaluating Arguments

Recall that logic is a science that aims to separate good arguments from bad arguments. Tagging an argument as "good" or "bad" involves evaluating the *quality* of an argument. What makes an argument good? This partly depends upon the *purpose* of the argument. If we only expect arguments to entertain us or be thought-provoking, then whether the premises of the argument are true or not might not be of our concern. If we only want arguments to consist of only true propositions, then we might read-off each proposition of the argument and determine whether it is true (or false). That is, we would evaluate the argument independent of the *relation* between the premises and the conclusion. If, however, we want *the argument to be truth-preserving* (viz., it is impossible for the premises to be true and the conclusion to be false), then our focus is principally on the *relationship between the premises and the conclusion.* This latter desire is the one pursued in this text.

1.3 Deductive Validity

> **Definition – Deductive Validity**
>
> An argument is deductively valid if and only if it is logically impossible for the premises to be true and the conclusion false. Alternatively put, an argument that is deductively valid is one where it is necessarily the case that if the premises are true, then the conclusion is true.

So, a valid argument that has all true premises must have a true conclusion. An initial confusion involved in understanding the concept of deductive validity is that people sometimes think that in order for an argument to be deductively valid, the premises and the conclusion must, in fact, be true. This is not the case for when we consider whether an argument is deductively valid, we don't consider the actual truth or falsity of the premises but rather consider whether it is logically impossible for all of the premises to be true and the conclusion false. What this means then is that an argument can be deductively valid where the argument has:

1. true premises and a true conclusion
2. false premises and a true conclusion
3. false premises and a false conclusion

What cannot be the case is an argument that has true premises and a false conclusion. In fact, that scenario cannot be possible.

To see this more clearly, consider that the propositions in an argument (the premises and the conclusion) can be true or false. Let's represent some of these combinations as follows:

Premise	True	True	False	True	True	False
Premise	True	False	False	True	False	False
Conclusion	True	True	True	False	False	True
	1	2	3	4	5	6

Table 1.3 – Argument is valid if and only if column 4 is impossible

An argument is deductively valid if and only if column (4) is logically impossible. That is, that (4) cannot occur. To illustrate, consider the following argument:

P1 Either Jennifer Lopez or Mario Lopez is the president of the United State of America (USA)
P2 Mario Lopez is not the president of USA.
C Therefore, Jennifer Lopez is the president of USA.

Argument 1.3

Notice that premise (P1) is false. It is not the case that either Jennifer Lopez or Mario Lopez is the president of the USA. Nevertheless, if the premises (P1) and (P2) were true, would the conclusion also be true? The answer is a resounding Yes! It is impossible for (P1) and (P2) to be true while (C) is false. That is, it is necessarily the case that if (P1) and (P2) are true, then (C) is true. Thus, the argument is deductively valid.

1.3.1 The Imagination Test for Deductive Validity

How do we determine which arguments are valid and which are invalid? One answer is this: we determine which arguments are deductively valid (or invalid) by trying to imagine a scenario in which an argument's premises are true and its conclusion is false. If we are unable to imagine such a scenario, then the argument is valid. If we are able to imagine such a scenario, then the argument is invalid.

Let's call this method the "human-imagination method" for determining valid arguments. This method feeds arguments to human subjects and then asks the subjects to imagine whether they can imagine a scenario where the premises are true and the conclusion is false. If they can, then the argument is invalid. If they cannot, the argument is invalid.

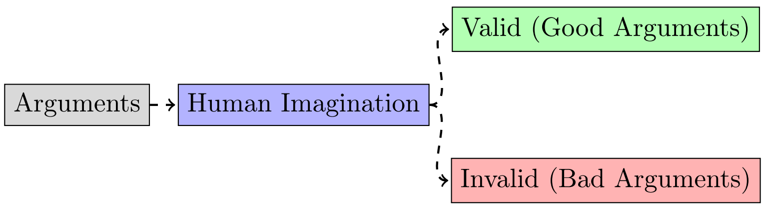

Figure 1.4 – Human-Imagination Method for Testing Arguments

How does the "human-imagination" method for testing arguments work? First, we consider the question of whether an argument is deductively valid in the following terms "Is it possible for all of the premises to be true and the conclusion false?" Second, we try to imagine a scenario where the premises of the argument are *true* and the conclusion is *false*. Third, if such a scenario is imaginable, then the human-imagination method says the argument is not valid (invalid). If such a scenario is not imaginable, then the human-imagination method says the argument is valid.

Let's consider a particular use of the human-imagination method on an argument considered earlier (see Argument 1.4).

P1	Either Jennifer Lopez or Mario Lopez is the president of the United State of America (USA)
P2	Mario Lopez is not the president of USA.
C	Therefore, Jennifer Lopez is the president of USA.

Argument 1.4 – An Argument for Jennifer Lopez as President

To determine whether this argument is valid or invalid, we can first ask whether we can imagine a scenario where all of the premises to be true (P1 and P2) and the conclusion is false (C). If we can imagine such a scenario, then the argument is *invalid*. If we cannot imagine such a scenario, then the argument is *valid*.

Let's consider another example (see Argument 1.5).

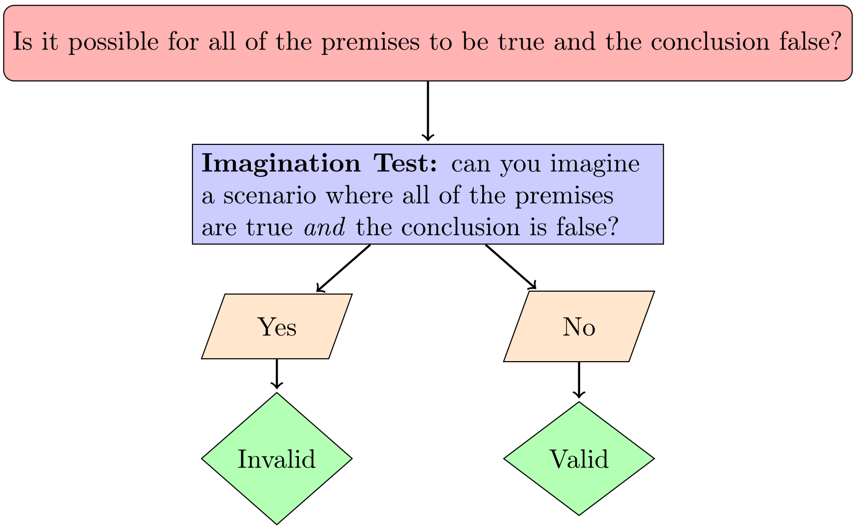

Figure 1.5 – Imagination Test for Valdiity

P1	Some basketball players are millionaires.
P2	Some millionaires have fancy cars.
C	Therefore, some basketball players have fancy cars.

Argument 1.5 – An Argument for Basketball Players Having Fancy Cars

	QUESTION	RESULT
(1)	Is it possible for all of the premises to be true?	Yes!
(2)	If the answer to (1) is yes, then is it logically possible for all of the premises to be true and the conclusion false?	Yes!
3	If the answer to (2) is yes, then the argument is not valid.	Not Valid!

Table 1.4 – Imagination test of Argument 1.5

It is logically possible for all of the premises to be true and the conclusion false because it is possible that basketball players do not have fancy cars, i.e., they may buy large homes and fancy jewelry but own modest cars.

1.4 Soundness

DEFINITION – SOUND

An argument is sound if and only if all of the premises are (in fact) true and it is deductively valid.

In short, soundness = valid + all true premises. We won't be concerned with soundness in this course, but is important to know that soundness is a stronger notion than validity since an argument can be valid without being sound.

Exercise 3: *Valid or Invalid?*

1. All pigs fly. Babe is a pig. Therefore, Babe flies.
2. John eats brains if and only if John is a zombie. John is not a zombie. Therefore, John does not eat brains.
3. John is happy. Therefore, John is happy or hungry.
4. God exists or he doesn't. Well, I see no good reason for thinking God exists. Therefore, God doesn't exist.

1.5 Problems with the Imagination Test

Objection 1. Too Many Premises. First, using the "imagination method" seems relatively straightforward when dealing with arguments that are composed of one or two premises, but what about arguments with nine, ten, fifty, or a hundred premises. In other words, we can imagine feeding P_1, P_2, P_3 to a human subject and it being capable of telling us whether or not it is a good or bad argument.

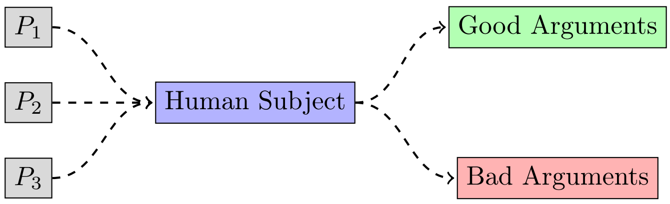

Figure 1.6 – Imagination Method for Testing Arguments

But, at some point, human beings will struggle with imagining whether it is possible for some large, but finite, number of premises $A_1, A_2, A_3 \ldots A_n$ is true and the conclusion false. That is, at some point, human beings will become exhausted and no longer be able to tell us whether or not the argument is good or bad. In short, it would be nice if there were some way to evaluate whether or not an argument is good or bad that does not depend upon the finite powers of human beings.

Objection 2. Human Bias & Disagreement. A second problem with the human-imagination method is that it is unclear how human subjects are evaluating the arguments they are given. Subjects may evaluate an argument as a "good argument" or "correct reasoning" when it is in conflict with the facts simply because they desire the conclusion to be true.

In short, it would be nice if there were some way to evaluate arguments using certain criteria that is as independent of human bias.

Objection 3. Ambiguity A third problem with the human-imagination method is that human beings evaluate arguments in a natural language that allows for different forms of imprecision. For example, arguments that have ambiguous terms allow for equivocation.

P1	A pencil is light
P2	Light has no weight
C	Therefore, a pencil has no weight.

Argument 1.6 – Argument for the weightlessness of pencils.

In Argument 1.6, “light” is used in two different ways: (i) not being heavy, and (ii) a kind of electromagnetic radiation that the human eye can see.

P1	Every student in logic did not pass the exam.
P2	John is a student in logic.
C	Therefore, John did not pass the exam.

Argument 1.7 – Argument for the weightlessness of pencils.

(P1) is ambiguous. If (P1) means “no student in logic passed the exam”, then Argument 1.7 is valid. If, however, (P1) means “there exists at least one student who didn’t pass the logic exam”, then Argument 1.7 is invalid.

Questions

1. What is logic?
2. What is an argument?
3. What is a proposition?
4. Does the following sentence express a proposition: There are ten thousand atoms in the universe.
5. Does the following sentence express a proposition: Clean your room!
6. Does the following sentence express a proposition: Oh my!
7. Does the following sentence express a proposition: What is the answer to question 5?
8. What types of sentences do not express propositions?
9. How is an argument different from arguing?
10. What does it mean to say that an argument is *deductively valid*?
11. How can we test arguments to determine if they are deductive valid
12. What are some problems associated with the “human-imagination” test for validity?
13. What does it mean to say that an argument is sound?
14. Can an argument be sound if it is not valid?
15. Can an argument be sound if it has at least one false premise?
16. Is it possible for an argument to be valid, have true premises and a false conclusion?
17. Is it possible for an argument to be valid yet have false premises?
18. Is it possible for an argument to be valid, have one true premise and one false premise?
19. Assuming that an argument is valid and has true premises, is it possible for that argument to have a false conclusion?
20. Are valid arguments truth-preserving? That is, if the premises of a valid argument are true, does this guarantee the truth of the conclusion?