

Three Common Mistakes on Quiz #5

David W. Agler
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Mistake #1: Abusing Disjunction Elimination ($\vee E$)

A very common mistake on Quiz #5 was this:

1	$P \vee Q$	P
2	P	1$\vee E$ – NO!

In English, what the above says is the following:

1	John is happy or Mary is a queen.	P
2	Therefore, John is happy.	1$\vee E$ – NO!

The above argument is clearly invalid since it is possible for (1) to be true and (2) to be false.

Disjunction elimination ($\vee E$) is actually a much more complicated rule. It involves a disjunction, making two assumptions, and reasoning to the same proposition within two subproofs.

If you use $\vee E$, your use should look something like this:

1	$P \vee Q$	P
2		P
3		.
4		.
5		R
6		Q
7		.
8		.
9		R
10	R	1,2-5, 6-9$\vee E$

Mistake #2: Using Conjunction Elimination ($\wedge E$) on Propositions that are NOT Conjunctions

Take a look at the following proof:

1	$(P \wedge R) \rightarrow Z$	P
2	P	1$\wedge E$ – NO!

Conjunction elimination can only be applied on propositions that are *conjunctions*! While line (1) has a conjunction as the antecedent of the conditional, it is not a conjunction. Line (1) is a conditional. A correct use of $\rightarrow E$ looks like this:

1	$(P \wedge R) \rightarrow Z$	P
2	$P \wedge Z$	P
3	P	2\wedgeE – YES!
4	Z	2\wedgeE – YES!

Mistake #3: Using Conjunction Elimination ($\rightarrow E$) without the Antecedent of the Conditional

Take a look at the following proof:

1	$P \rightarrow Z$	P
2	P	1\rightarrowE – NO!
3	Z	1\rightarrowE – NO!

In English, what this says is the following:

1	If John is happy, then he'll eat his birthday cake.	P
2	From (1), it follows that John is happy.	1\rightarrowE – NO!
3	From (1), it follows that John will eat his birthday cake..	1\rightarrowE – NO!

Since it possible for (1) to be true and (2) false, and possible for (1) to be true and (3) false, reasoning like the above is invalid.

In order to use $\rightarrow E$ correctly, you need two propositions, a conditional $P \rightarrow R$ and the antecedent of that conditional **P**. If you have both of these propositions, then you can derive the consequent of that conditional **R**.

1	If John is happy, then he'll eat his birthday cake.	P
2	John is happy.	P
3	From (1) and (2), it follows that John will eat his birthday cake.	1,2\rightarrowE

Another example of the correct use of $\rightarrow E$ looks like this:

1	$(P \wedge R) \rightarrow Z$	P
2	P	P
3	R	P
4	$P \wedge R$	2,3 \wedge I
5	Z	1,4\rightarrowE – YES!