HANDOUT #3R, **REVIEW** OF LESSONS 1, 2, & 3

Literal Negation & Two Style Conventions

To form the literal negation of a proposition **P**, simply put parentheses around **P** and then put a negation to the left of **(P)**.

Proposition	Add Parentheses	Add Negation to the Left
P∧R	(P∧R)	¬(P ∧R)
¬P∧R	(¬P∧R)	¬(¬P∧R)
(P∧M)↔S	((P∧M)↔S)	¬((P∧M)↔S)
¬((¬P∨¬M)↔¬S)	(¬((¬P∨¬M)↔¬S))	¬(¬((¬P∨¬M)↔¬S))

One convention (Convention #1) is to remove a set of parentheses if the sign for negation applies to a single propositional letter or if it applies to a negated proposition.

Proposition	Add Parentheses	Add Negation to the Left	Convention #1
P	(P)	¬(P)	¬P
¬P	(¬P)	¬(¬P)	¬¬P
¬((¬P∨¬M)↔¬S)	(¬((¬P∨¬M)↔¬S))	¬(¬((¬P∨¬M)↔¬S))	¬¬((¬P∨¬M)↔¬S)

A second convention (Convention #2) that is employed is to use braces [ ] and brackets { } when there is a set of parentheses is contained in another set of parentheses.

Proposition	Convention #2
¬((P∧M)↔S)	¬[(P∧M)↔S]
¬(¬((¬P∨¬M)↔¬S))	¬¬[(¬P∨¬M)↔¬S)]
P∨(S∨(M∨(L∧T)))	P∨{S∨[M∨(L∧T)]}

Exercise Set #1: Determine the literal negation of the following propositions
1. P
2. P∧¬R
3. ¬¬P∧¬R
4. ¬(¬P∧¬R)
5. [¬P→(¬S∧M)]→L

Scope & Main Operator

There are two types of operators: *the connectives* ∧, ∨, →, ↔ and the sign for negation ¬. The connectives operate upon the propositions to their left and right. The sign for negation applies to the proposition to its immediate right. Operators can operate upon propositional letters or complex propositions. When an operator operates upon propositional letters or a negated propositional letter, no parentheses are needed.

Proposition
¬P
P∧R
P∨R
P→R
P↔R
¬P∧R
P∨¬R
¬P→¬R
¬P↔¬R

Parentheses are required when we want any *operator* to apply to a complex proposition that involves a *connective*.

NO!	YES!
P∧R∨S	P∧(R∨S)
P∧R∨S	(P∧R)∨S
¬P∧R→S	¬P∧(R→S)
¬P∧R→S	¬(P∧R)→S
¬P∧R→S	¬[P∧(R→S)]
¬P∧R→S	¬[(P∧R)→S]

An operator's scope pertains to what formula it operates upon. For example, the scope of '∧' in 'P∧R' is on 'P' and 'R'. Operators can have narrow scope or wide scope. This is determined by the use of parentheses or certain conventions we have been using.

Formula	Scope
¬P∧R	Negation has narrow scope, it only applies to P
¬(P∧R)	Negation has wide scope, it applies to P∧R
P∧(R∨S)	∧ has wide scope, it applies to P and R∨S
(P∧R)∨S	∨ has wide scope, it applies to P∧R and S

The **main operator** is the operator with the most scope. It has all other operators within its scope.

Formula	Main Operator
¬P∧R	∧ is the main operator
¬(P∧R)	¬ is the main operator
P∧(R∨S)	∧ is the main operator
(P∧R)∨S	∨ is the main operator

Exercise Set #2: Determine the main operator of the following formula

1. ¬¬P
2. ¬¬P∧¬R
3. ¬(¬P∧¬R)
4. [¬P→(¬S∧M)]→L

Types of Propositions

You want to become comfortable with *what to call* these types of formula:

Formula	Type of Proposition
P	Propositional Letter or Atomic Proposition
$\neg$P	Negated Proposition
$\neg\neg$P	Doubly-Negated Proposition
P$\wedge$R	Conjunction
P$\vee$R	Disjunction
P$\rightarrow$R	Conditional
P$\leftrightarrow$R	Biconditional
$\neg$(P$\wedge$R)	Negated Conjunction
$\neg$(P$\vee$R)	Negated Disjunction
$\neg$(P$\rightarrow$R)	Negated Conditional
$\neg$(P$\leftrightarrow$R)	Negated Biconditional

Exercise Set #3: Determine the type of proposition of the following formula

1. $P\wedge\neg R$
2. $\neg P\wedge\neg R$
3. $\neg(\neg P\wedge\neg R)$
4. $\neg\neg(P\wedge R)$
5. $\neg\neg\neg\neg\neg P\rightarrow\neg(\neg R\wedge\neg S)$

Truth Table Construction

Creating Truth Tables is a Three-Step Process:

Step #1 (Setting it Up), *Represent all of the different ways you can assign truth values (T & F) to the propositional letters.*

P	R	P	$\rightarrow$	$\neg$	(R	$\wedge$	P)
T	T						
T	F						
F	T						
F	F						

Step #2 (Transfer the Data)*: For each row, write the truth value assigned to a given propositional letter under that letter in the formula.*

P	R	P	$\rightarrow$	$\neg$	(R	$\wedge$	P)
T	T	T			T		T
T	F	T			F		T
F	T	F			T		F
F	F	F			F		F

Step #3 (Determine the Truth Value of the Proposition)*: Starting with the operator with the least scope (or one that applies to propositional letters), determine the truth value of the wff upon which that operator; then proceed to the operator with the next least amount of scope.*

P	R	P	→	¬	(R	∧	P)
T	T	T			T	**T**	T
T	F	T			F	**F**	T
F	T	F			T	**F**	F
F	F	F			F	**F**	F

Notice that we can determine the truth value of 'R∧P' since we know the truth values of 'R' and 'P'. *We can now forget about the truth value of R* and *P since all we need to know is the truth value of R∧P to determine the truth value of ¬(R∧P)*

P	R	P	→	¬	(R	∧	P)
T	T	T		**F**		T	
T	F	T		**T**		F	
F	T	F		**T**		F	
F	F	F		**T**		F	

Next, we can determine the truth value of ¬(R∧P) since we know the truth value of R∧P. *We can now forget about the truth-value* of (R∧P) since all we need to know is the truth value of ¬(R∧P) to determine the truth value of P→¬(R∧P)

P	R	P	→	¬	(R	∧	P)
T	T	T	**F**	F			
T	F	T	**T**	T			
F	T	F	**T**	T			
F	F	F	**T**	T			

The truth value under the main operator determines the truth value of the entire wff given a truth-value assignment.

Exercise Set #4: Create a Truth Table for the following wffs
1. ¬P∧¬R
2. ¬P→¬R
3. ¬(P→¬R)
4. [(P∧¬R)]→¬R
5. ¬[(P∧¬R)→(¬R↔P)]