Directions: This exam has 22 questions, for a total of 100 points and 30 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

Multiple Choice

Directions: Answer the questions in the spaces provided by circling one and only one answer (unless the question states otherwise).

1. (3 points) What is a deductive apparatus for **RL**?
   A. a set of rules of derivation that express which wffs ϕ can be written after which wffs ψ in a derivation.
   B. a set of rules that state how a tree is supposed to look, e.g. horizontally rather than vertically.
   C. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.
   D. a set of rules that state that the rows in a proof need to be numbered.
   E. a way of listing off each wff one right after another

2. (3 points) What is a derivation of **Q** using **RD**?
   A. a finite string of formulas from a set Γ of **RL** wffs where (i) the last formula in the string is Q and (ii) each formula is either a premise, an assumption, or is the result of the preceding formulas and the deductive apparatus.
   B. finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with **Q**.
   C. a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.
   D. an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.

3. (3 points) What does the following mean: $\Gamma \vdash \mathbf{Q}$
   A. $\Gamma \vdash \mathbf{Q}$ says **Q** is a *syntactic* consequence of Γ (meaning that there is a derivation of **Q** from Γ).
   B. $\Gamma \vdash \mathbf{Q}$ says **Q** is a *semantic* consequence of Γ (there is no model such that the wffs of Γ are true and **Q** is false).
   C. $\Gamma \vdash \mathbf{Q}$ says **Q** is a *hypostatic abstraction* from Γ
   D. $\Gamma \vdash \mathbf{Q}$ says **Q** intuitively follows from Γ. That is, if you imagine Q in a proof, you can reason to Γ.

Directions: Answer the questions on the line provided by writing the abbreviation for the derivation rule that is best described in the question prompt provided.

4. (3 points) What derivation rule best describes the following reasoning: Everyone is angry. Therefore, Liz is angry.

5. (3 points) What derivation rule best describes the following reasoning: Tek loves everyone. Therefore, Tek loves Liz.

6. (3 points) What derivation rule is best described as follows: from the existentially quantified wff $(\exists x)\phi(x)$ and a subproof that begins with an assumption $\phi(\alpha/x)$, a wff ψ may be derived provided (i) the name α is foreign to the proof and (ii) ψ does not contain α.

7. (3 points) What derivation rule is best described as follows: given a wff Za, a universally quantified wff can be derived (e.g., $(\forall x)Zx$) provided (i) a does not occur as a premise or as an assumption in an open subproof, and (2) a does not occur in $(\forall x)Zx$.

8. (3 points) What single derivation rule would allow you to reason to Qb from $(\exists x)Rx \wedge Qb$?

9. (3 points) What single derivation rule would allow you to reason to Lbb from $(\forall x)Lxx$?

10. (3 points) What single derivation rule would allow you to reason to $(\exists x)Lxx$ from Laa?

11. (3 points) What single derivation rule would allow you to reason to $(\forall x)\neg(Px \vee Qx)$ from $\neg(\exists x)(Px \vee Qx)$?

12. (3 points) What single derivation rule would allow you to reason to $Pa \to Qa$ from $(\forall x)(Px \to Qx)$?

13. (3 points) Assuming "a" is not present in an active assumption nor a premise, what single derivation rule would allow you to derive $(\forall x)(Lx \to Mx)$ from $La \to Ma$?

14. (1 point) Free point because I need the exam to be out of 100.

Derivations

Directions: Solve the following proofs.

15. (10 points) $Pa, Qa, (\exists x)(Px \wedge Qx) \to (\forall x)(Mx \wedge Lx) \vdash Mb$

16. (10 points) $Pab, (\exists x)(\exists y)Pxy \to (\forall x)Zxx \vdash Zaa$

17. (10 points) $(\exists x)(Px \wedge Qx) \vdash (\exists x)Qx$

18. (10 points) $(\forall x)(\forall y)Lxy \vdash (\exists y)(\exists x)Lxy$

19. (10 points) $\vdash (\exists x)(Px \vee \neg Px)$

20. (10 points) $\vdash (\forall x)(Px \to (Qx \to Px))$

Bonus

21. (15 points (bonus)) Prove $(\exists x)\neg Px \dashv\vdash \neg(\forall x)Px$ without using QN

22. (15 points (bonus)) Prove $\neg(\exists x)Px \dashv\vdash (\forall x)\neg Px$ without using QN

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$
$P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination $(\wedge E)$

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction $(\to I)$

n	$\quad$ | P	A
$\vdots$	$\quad$ | $\vdots$	
$(n+1)$	$\quad$ | Q	
$(n+2)$	$P \to Q$	$\to I$, n–$(n+1)$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

n	$\quad\vert\ \underline{P}$	A
$\vdots$	$\quad\vert\ \vdots$	
$(n+1)$	$\quad\vert\ Q$	
$(n+2)$	$\quad\vert\ \neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Derivation Rule – Negation Elimination ($\neg E$)

n	$\quad\vert\ \underline{\neg(P)}$	A
$\vdots$	$\quad\vert\ \vdots$	
$(n+1)$	$\quad\vert\ Q$	
$(n+2)$	$\quad\vert\ \neg Q$	
$(n+3)$	P	$\neg E$, n–$(n+2)$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

1	$P \vee Q$	P
n	$\quad\vert\ \underline{P}$	A
$\vdots$	$\quad\vert\ \vdots$	
$(n+1)$	$\quad\vert\ R$	
(i)	$\quad\vert\ \underline{Q}$	A
$\vdots$	$\quad\vert\ \vdots$	
$(i+1)$	$\quad\vert\ R$	
(k)	R	$\vee E$, 1, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
(i)	Q	A
$\vdots$	$\vdots$	
$(i+1)$	P	
(k)	$P \leftrightarrow Q$	$\leftrightarrow I$, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \vee Q, \neg Q \vdash P$ or $P \vee Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \rightarrow Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$

Derivation Rule – Implication (IMP)

$P \rightarrow Q \dashv\vdash \neg P \vee Q$

Derivation Rule – Universal Elimination ($\forall E$)

$(\forall x)P \vdash P(a/x)$

Derivation Rule – Existential Introduction ($\exists I$)

$Pa_n \vdash (\exists x)P(x_n/a_n)$

Derivation Rule – Universal Introduction ($\forall I$)

$Pa_n \vdash (\forall x)P(x_n/a_n)$ (when a does not occur as premise, as an assumption in an open subproof, or in $(\forall x)P$).

Derivation Rule – Existential Elimination ($\exists E$)

1	$(\exists x)\mathbf{P}$	P
n	$\mathbf{P}(a/x)$	A
$\vdots$	$\vdots$	
$(n+1)$	$\mathbf{Q}$	
(k)	$\mathbf{Q}$	$\exists E$, 1, n–$(n+1)$

Derivation Rule – Quantifier Negation (QN)

$\neg(\forall x)P \dashv\vdash (\exists x)\neg P$
$\neg(\exists x)P \dashv\vdash (\forall x)\neg P$

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Directions: Please write your **name** on the top of this page. Answer all of the questions on the answer sheet provided. If an answer will not fit on the blank provided, place your answer on one of the several blank pages.

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