Directions: This exam has 19 questions, for a total of 99 points and 0 bonus points. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may use the last pages of this exam as scrap paper.

Short Answer

Directions: Answer the questions on the line provided by writing the abbreviation for the derivation rule that is best described in the question prompt provided.

1. (3 points) What single derivation rule would allow you to reason to $(\exists x)Qx$ from Qb?
2. (3 points) What single derivation rule would allow you to reason to Mb from $Qb \to Mb, Qb$?
3. (3 points) What single derivation rule would allow you to reason to $\neg Rbb$ from $(\forall x)\neg Rxx$?
4. (3 points) What single derivation rule would allow you to reason to $\neg(\forall x)Lxx$ from $(\exists x)\neg Lxx$
5. (3 points) What derivation rule is best described as follows: given an existentially quantified wff $(\exists x)Px$, we can infer a wff ϕ by assuming a wff of the form Pa provided (i) a is not found in a premise or any other active part of the proof and (ii) a is not found in ϕ.
6. (3 points) What derivation rule is best described as follows: given a wff Pa, a universally quantified wff can be derived (e.g., $(\forall x)Px$) provided (i) a does not occur as a premise or as an assumption in an open subproof, and (2) a does not occur in $(\forall x)Px$.
7. (3 points) What derivation rule best describes the following reasoning: Tek is the best. Therefore, someone is the best.
8. (3 points) What derivation rule best describes the following reasoning: Everyone loves themselves. Therefore, Tek loves himself.
9. (3 points) What derivation rule best describes the following reasoning: Tek is sad and Tek is angry. Therefore, someone is both sad and angry.
10. (3 points) What derivation rule best describes the following reasoning: it is not the case that everyone is happy. Therefore, someone is not happy.

Conceptual

11. (3 points) What is a deductive apparatus for **RL**?
    A. a set of rules that state that the rows in a proof need to be numbered.
    B. a set of rules that state how the proof is supposed to look, e.g. horizontally rather than vertically.
    C. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.
    D. a set of rules of derivation that express which wffs $\mathbf{Q}$ can be written after which wffs $\mathbf{P}$ in a derivation.
12. (3 points) What is a derivation of $\mathbf{Q}$ using **RD**?
    A. a finite string of formulas from a set Γ of **RL**wffs where (i) the last formula in the string is Q and (ii) each formula is either a premise, an assumption, or is the result of the preceding formulas and the deductive apparatus.
    B. finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with $\mathbf{Q}$.

C. a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with $\mathbf{Q}$.

D. an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with $\mathbf{Q}$.

13. (3 points) What is the difference between $\Gamma \vdash \mathbf{Q}$ and $\Gamma \models \mathbf{Q}$?

A. $\Gamma \vdash \mathbf{Q}$ says $\mathbf{Q}$ is a *syntactic* consequence of Γ (meaning that there is a derivation of $\mathbf{Q}$ from Γ). In contrast, $\Gamma \models \mathbf{Q}$ says that $\mathbf{Q}$ is a *semantic* consequence of Γ, which means that there is no model such that the wffs of Γ are true and $\mathbf{Q}$ is false.

B. $\Gamma \vdash \mathbf{Q}$ says $\mathbf{Q}$ is a *semantic* consequence of Γ (there is no model such that the wffs of Γ are true and $\mathbf{Q}$ is false). In contrast, $\Gamma \models \mathbf{Q}$ says that $\mathbf{Q}$ is a *syntactic* consequence of Γ (meaning that there is a derivation of $\mathbf{Q}$ from Γ).

Essay

Directions: Solve the following proofs.

14. (10 points) $Pa, Qb \vdash (\exists x)Px$

15. (10 points) $(\forall x)(Qx \wedge Px) \vdash (\exists x)(Px \wedge Qx)$

16. (10 points) $(\forall x)(Px \wedge Qx) \vdash Qc \wedge Qb$

17. (10 points) $(\exists x)Px \vdash (\exists y)(Py \vee Ly)$

18. (10 points) $(\forall x)(Px \rightarrow Qx), (\forall x)(Qx \rightarrow Mx), (\forall x)Px \vdash (\forall x)Mx$

19. (10 points) $\vdash (\forall x)((Mx \wedge Qx) \rightarrow Qx)$

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$
$P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction ($\rightarrow I$)

$$\begin{array}{lll}
n & \quad\vert\ \underline{P} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ Q & \\
(n+2) & P \rightarrow Q & \rightarrow I,\ n\text{–}(n+1)
\end{array}$$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

$$\begin{array}{lll}
n & \quad\vert\ \underline{P} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ Q & \\
(n+2) & \quad\vert\ \neg Q & \\
(n+3) & \neg(P) & \neg I,\ n\text{–}(n+2)
\end{array}$$

Derivation Rule – Negation Elimination ($\neg E$)

$$\begin{array}{lll}
n & \quad\vert\ \underline{\neg(P)} & \text{A} \\
\vdots & \quad\vert\ \vdots & \\
(n+1) & \quad\vert\ Q & \\
(n+2) & \quad\vert\ \neg Q & \\
(n+3) & P & \neg E,\ n\text{–}(n+2)
\end{array}$$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

$$
\begin{array}{lll}
1 & P \lor Q & \text{P} \\
n & \quad | \ \underline{P} & \text{A} \\
\vdots & \quad | \ \vdots & \\
(n+1) & \quad | \ R & \\
(i) & \quad | \ \underline{Q} & \text{A} \\
\vdots & \quad | \ \vdots & \\
(i+1) & \quad | \ R & \\
(k) & R & \lor E,\ 1,\ n\text{–}(n+1),\ (i)\text{–}(i+1)
\end{array}
$$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

$$
\begin{array}{lll}
n & \quad | \ \underline{P} & \text{A} \\
\vdots & \quad | \ \vdots & \\
(n+1) & \quad | \ Q & \\
(i) & \quad | \ \underline{Q} & \text{A} \\
\vdots & \quad | \ \vdots & \\
(i+1) & \quad | \ P & \\
(k) & P \leftrightarrow Q & \leftrightarrow I,\ n\text{–}(n+1),\ (i)\text{–}(i+1)
\end{array}
$$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \lor Q, \neg Q \vdash P$ or $P \lor Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \to Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \to Q, Q \to R \vdash P \to R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \lor Q) \dashv\vdash \neg P \land \neg Q$
$\neg(P \land Q) \dashv\vdash \neg P \lor \neg Q$

Derivation Rule – Implication (IMP)

$P \to Q \dashv\vdash \neg P \lor Q$

Derivation Rule – Universal Elimination ($\forall E$)

$(\forall x)P \vdash P(a/x)$

Derivation Rule – Existential Introduction ($\exists I$)

$Pa_n \vdash (\exists x)P(x_n/a_n)$

Derivation Rule – Universal Introduction ($\forall I$)

$Pa_n \vdash (\forall x)P(x_n/a_n)$ (when a does not occur as premise, open subproof, or in $(\forall x)P$).

Derivation Rule – Existential Elimination ($\exists E$)

1	$(\exists x)\mathbf{P}$	P
n	$\quad\mid\ \mathbf{P}(a/x)$	A
$\vdots$	$\quad\mid\ \vdots$	
$(n+1)$	$\quad\mid\ \mathbf{Q}$	
(k)	$\mathbf{Q}$	$\exists E$, 1, n–$(n+1)$

Derivation Rule – Quantifier Negation (QN)

$\neg(\forall x)P \dashv\vdash (\exists x)\neg P$
$\neg(\exists x)P \dashv\vdash (\forall x)\neg P$

Evaluation

Page:	1	2	Total
Points:	36	63	99
Bonus Points:	0	0	0
Score:			

Directions: Please write your **name** on the top of this page. Answer all of the questions on the answer sheet provided. If an answer will not fit on the blank provided, place your answer on one of the several blank pages.

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