Directions: This exam has 26 questions, for a total of 100 points and 0 bonus points. Write your **name**, the **exam version**, and your **answers** on the answer sheet provided. Please read the directions for each section carefully. If you have any questions about the exam itself, please raise your hand and I will come to your desk to answer your question. You may write on this exam and may use the last pages of this exam as scrap paper.

Multiple Choice: Concepts and terminology

1. (2 points) What is a derivation of **Q** from **Γ** using **PD**?
   A. A derivation of **Q** is a *finite* string of formulas from a set Γ of **PL** wffs where (i) the last formula in the string is **Q** and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.
   B. A derivation of **Q** is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and ending with **Q**.
   C. A derivation of **Q** is a finite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.
   D. A derivation of **Q** is an *infinite* string of formulas from a set Γ of **PL** wffs where (i) the last formula in the string is **Q** and (ii) each wff in the set is either a premise, an assumption, or is the result of the preceding wffs and the deductive apparatus.
   E. A derivation of **Q** is an infinite string of wffs starting with some premises $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ or assumptions and ending with **Q**.

2. (2 points) What is a deductive apparatus for **PL**?
   A. a set of rules that state how the proof is supposed to look, e.g. horizontally rather than vertically.
   B. It is a set of rules that allow individuals to reason from facts (experience) to general laws, e.g. laws of nature.
   C. It is a set of rules of reason that all people use to reason from one proposition to another, including, but not limited to, hypothetical and probabilistic reasoning.
   D. a set of rules that state that the rows in a proof need to be numbered.
   E. a set of derivation rules that express which wffs ϕ can be written after which wffs ψ in a derivation.

3. (2 points) In logic, there are two notions of logical consequence (entailment). The first notion is the semantic (model-theoretic) notion, which says that Q is a logical consequence of Γ if and only if there is no interpretation of the members of Γ and Q such that all of the members of Γ are true and Q is false. What is the other notion of logical consequence?
   A. the semi-semantic notion of logical consequence (entailment). This notion says that Q is a semi-semantic consequence of Γ if and only if it is impossible for the premises to be true and the conclusion false, defined in terms of truth tables and trees.
   B. the intuitive notion of logical consequence (entailment). This notion says that Q is a *logical* consequence of Γ if and only if Q intuitively follows from Γ
   C. the legal notion of logical consequence (entailment). This notion says that Q is a logical consequence of Γ if and only if Q would be accepted in a court of law or some practical matter.
   D. the syntactic (proof-theoretic) notion of logical consequence (entailment). This notion says that Q is a logical consequence of Γ if and only if there is a derivation of Q from Γ
   E. none of the above

4. (2 points) What is the difference between $\Gamma \vdash \mathbf{Q}$ and $\Gamma \models \mathbf{Q}$?

A. $\Gamma \vdash \mathbf{Q}$ is syntactic consequence while $\Gamma \models \mathbf{Q}$ is semantic consequence.

B. $\Gamma \models \mathbf{Q}$ is syntactic consequence while $\Gamma \vdash \mathbf{Q}$ is semantic consequence.

Short Answer

Directions: Answer the questions on the line provided on the answer sheet by writing the abbreviation for the derivation rule (e.g. $\leftrightarrow E$ that is best described in the question prompt provided.

5. (2 points) What single derivation rule would allow you to reason to $S \wedge Q$ from $(S \wedge Q) \wedge M$
6. (2 points) What single derivation rule would allow you to reason to $Z \wedge (Q \vee \neg P)$ from Z and $Q \vee \neg P$?
7. (2 points) What single derivation rule would allow you to reason to $Z \rightarrow Q$ from $(Z \rightarrow Q) \leftrightarrow M$ and M
8. (2 points) What single derivation rule would allow you to reason to S from $\neg L \rightarrow S$ and $\neg L$
9. (2 points) What single derivation rule would allow you to reason to $M \vee \neg(Q \vee T)$ from M
10. (2 points) What single derivation rule would allow you to reason to $Q \rightarrow M$ from $Q \rightarrow \neg Z$ and $\neg Z \rightarrow M$
11. (2 points) What single derivation rule would allow you to reason to $\neg\neg(Q \vee T)$ from $Q \vee T$
12. (2 points) What single derivation rule would allow you to reason to $L \rightarrow M$ from $L \rightarrow Q, Q \rightarrow M$?
13. (2 points) What single derivation rule would allow you to reason to C from $C \leftrightarrow Q$ and Q
14. (2 points) What single derivation rule would allow you to reason to $M \wedge W$ from M and W
15. (2 points) What single derivation rule would allow you to reason to V from $V \vee P$ and $\neg P$
16. (2 points) What derivation rule is best described as follows: if P is assumed and within this subproof Q and $\neg(Q)$ is derived, then $\neg(P)$ can be derived.
17. (2 points) What derivation rule is best described as follows: if $(P \rightarrow Q) \wedge Q$ is on a line of the proof, then it is legitimate to derive $P \rightarrow Q$ on a line and it is legitimate to derive Q on another line.
18. (2 points) What derivation rule best describes the following reasoning: If John watches Netflix, then Mary will go to the party. John watches Netflix. Therefore, Mary will go to the party.
19. (2 points) What derivation rule is best described as follows: given $(X \wedge Y) \vee M$ and $\neg M$, then $X \wedge Y$ can be derived.
20. (2 points) What derivation rule is best described as follows: given $P \vee Q$, if P is assumed and it is shown that R follows from P and if Q is assumed and it is shown that R follows from Q, then R can be derived.

Derivations

Directions: Solve the following proofs. Be sure to setup the proof correctly, number all lines, and clearly indicate how each line is justified using the rules from the deductive apparatus.

21. (10 points) $(P \wedge \neg Q) \wedge R, M \wedge S \vdash M \wedge \neg Q$
22. (10 points) $(P \wedge Q) \rightarrow (R \wedge S), P, Q \vdash R$
23. (10 points) $\neg(P \vee Q) \vdash \neg P \leftrightarrow \neg Q$
24. (10 points) $\neg(M \rightarrow Q), S \rightarrow Q \vdash \neg S \vee T$
25. (10 points) $\vdash Q \rightarrow (B \vee \neg B)$
26. (10 points) $\vdash P \rightarrow (Q \rightarrow P)$

Derivation Rule – Conjunction Introduction $\wedge I$

$P, Q \vdash P \wedge Q$ or $P, Q \vdash Q \wedge P$

Derivation Rule – Conjunction Elimination ($\wedge E$)

$P \wedge Q \vdash P$ or $P \wedge Q \vdash Q$

Derivation Rule – Conditional Introduction ($\rightarrow I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$P \rightarrow Q$	$\rightarrow I$, n–$(n+1)$

Derivation Rule – Conditional Elimination ($\rightarrow E$)

$P \rightarrow Q, P \vdash Q$

Derivation Rule – Reiteration (R)

$P \vdash P$

Derivation Rule – Negation Introduction ($\neg I$)

n	P	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$\neg Q$	
$(n+3)$	$\neg(P)$	$\neg I$, n–$(n+2)$

Derivation Rule – Negation Elimination ($\neg E$)

n	$\neg(P)$	A
$\vdots$	$\vdots$	
$(n+1)$	Q	
$(n+2)$	$\neg Q$	
$(n+3)$	P	$\neg E$, n–$(n+2)$

Derivation Rule – Disjunction Introduction ($\vee I$)

$P \vdash P \vee Q$ or $P \vdash Q \vee P$

Derivation Rule – Disjunction Elimination ($\vee E$)

1	$P \lor Q$		P
n		P	A
$\vdots$		$\vdots$	
$(n+1)$		R	
(i)		Q	A
$\vdots$		$\vdots$	
$(i+1)$		R	
(k)	R		$\lor E$, 1, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Introduction ($\leftrightarrow I$)

n		P	A
$\vdots$		$\vdots$	
$(n+1)$		Q	
(i)		Q	A
$\vdots$		$\vdots$	
$(i+1)$		P	
(k)	$P \leftrightarrow Q$		$\leftrightarrow I$, n–$(n+1)$, (i)–$(i+1)$

Derivation Rule – Biconditional Elimination ($\leftrightarrow E$)

$P \leftrightarrow Q, P \vdash Q$ or $P \leftrightarrow Q, Q \vdash P$

Derivation Rule – Disjunctive Syllogism (DS)

$P \lor Q, \neg Q \vdash P$ or $P \lor Q, \neg P \vdash Q$

Derivation Rule – Modus Tollens (MT)

$P \to Q, \neg Q \vdash \neg P$

Derivation Rule – Hypothetical Syllogism (HS)

$P \to Q, Q \to R \vdash P \to R$

Derivation Rule – Double Negation (DN)

$P \dashv\vdash \neg\neg P$

Derivation Rule – De Morgan's Laws (DeM)

$\neg(P \lor Q) \dashv\vdash \neg P \land \neg Q$ or $\neg(P \land Q) \dashv\vdash \neg P \lor \neg Q$

Derivation Rule – Implication (IMP)

$P \to Q \dashv\vdash \neg P \lor Q$

Directions: Please write your **name** on the top of the page. Please write clearly. **J**

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