H8s: Further Explanation on Existential Elimination (∃E)

So here is a basic, no-nonsense explanation of ∃E put in pretty informal terms:

First, let's get the rule for ∃E out there:

	Existential Elimination (∃E)
	From an existentially quantified expression (∃x)**P**, an expression **Q** can be derived from the derivation of an assumed substitution instance **P**(a/x) of (∃x)**P** provided (1) the individuating constant *a* does *not* occur in *any* premise or in an active proof (or subproof) *prior* to its arbitrary introduction in the assumption **P**(a/x) and (2) the individuating constant *a* does not occur in proposition **Q** discharged from the subproof.
			(∃x)**P**
			    | **P**(a/x)
			    | .
			    | .
			    | .
			    | **Q**
			Q
				∃E

Let's focus on the first part of this rule. Later on, we will look at the second part. What ∃E says is that *starting* from an existentially quantified proposition, e.g. **(∃x)Px**, you can infer some other proposition **Q**.

(∃x)Px ← **All uses of ∃E start with an existentially quantified proposition**
    | Pa
    | .
    | .
    | .
    | **Q**
Q

But, you cannot reason directly from (∃x)Px to Q. You will have to go through a couple intermediate steps to get there. Let's look at those steps. First, all uses of ∃E require you to make an assumption, but not any assumption, an assumption where you remove the existential quantifier and replace bound variables with individual constants (names).

(∃x)Px
    | Pa ← **Need to make assumption based on the existentially quantified proposition**
    | .
    | .
    | .
    | **Q**
Q

Next, in that subproof, you will need to derive a proposition Q.

(∃x)Px
| Pa
| .
| .
| .
| **Q**
Q

Derive a proposition Q in the subproof using the derivation rules

Let's consider a real example of the above:

1	(∃x)Px		P
2		Pa	A
3		Pa∨Ra	2∨I
4		(∃x)(Px∨Rx)	3∃I
5	**Q**		

Now that you have derived a proposition **Q** within the subproof, you can rewrite that proposition outside of the subproof using ∃E.

(∃x)Px
| Pa
| .
| .
| .
| **Q**
Q

The proposition Q you derived in the subproof can be derived outside of the subproof by using ∃E

Continuing our example above:

1	(∃x)Px		P
2		Pa	A
3		Pa∨Ra	2∨I
4		(∃x)(Px∨Rx)	3∃I
5	(∃x)(Px∨Rx)		1, 2-4∃E

So, ∃E allows us to derive a proposition Q from an existentially quantified proposition (∃x)P. However, there are some restrictions on the use of ∃E. But what about the rest of the rule? Let's turn to the second half of ∃E now.

	Existential Elimination (∃E)
	From an existentially quantified expression (∃x)**P**, an expression **Q** can be derived from the derivation of an assumed substitution instance **P**(a/x) of (∃x)**P** provided (1) the individuating constant *a* does *not* occur in *any* premise or in an active proof (or subproof) *prior* to its arbitrary introduction in the assumption **P**(a/x) and (2) the individuating constant *a* does not occur in proposition **Q** discharged from the subproof.
			(∃x)**P**
			    | **P**(a/x)
			    | .
			    | .
			    | .
			    | **Q**
			Q
				∃E

The second half of ∃E places important restrictions on what can be assumed and the proposition derived outside of the subproof.

(∃x)Px
    | P(a/x)
    | .
    | .
    | .
    | **Q**
Q

Two restrictions

The first restriction states that if we want to use ∃E, we cannot assume a proposition that involves an individual constant (a – v) already in the proof. For example

1	(∃z)Wzz	P	
2	Wbb→Lc	P	**NO! b is already in the proof**
3	| Wbb	A	
4	| Lc	2,3→E	
5	Lc	1,3-4∃E	

The second restriction states that if you assume a proposition that involves an individual constant, you cannot derive a proposition that has *that* individual constant outside of the proof.

1	(∃z)Wzz	P	
2	| Wbb	A / ∃E	**NO! b was assumed at line 2**
3	| (∃x)Wbx	2∃I	
4	(∃x)Wbx	1, 2-3 ∃E — **NO!**	