

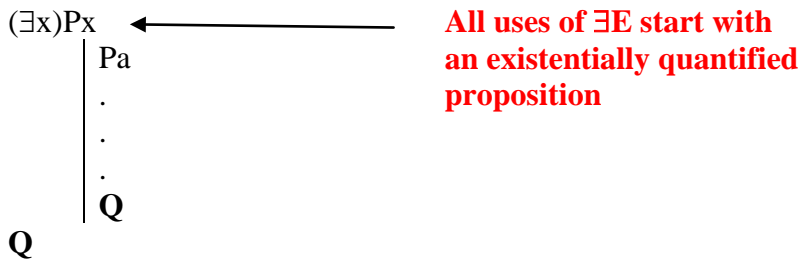
H8s: Further Explanation on Existential Elimination ($\exists E$)

So here is a basic, no-nonsense explanation of $\exists E$ put in pretty informal terms:

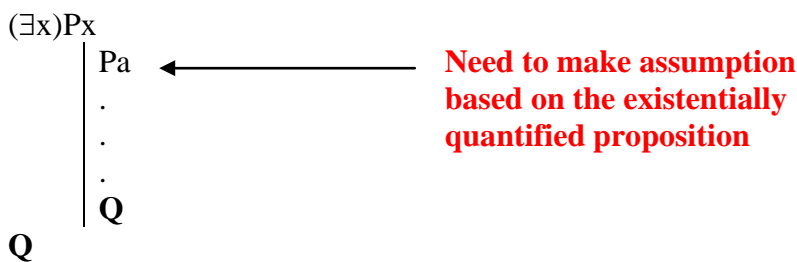
First, let's get the rule for $\exists E$ out there:

| | | |
|---|--|-------------|
| <p>Existential Elimination ($\exists E$) From an existentially quantified expression $(\exists x)P$, an expression Q can be derived from the derivation of an assumed substitution instance $P(a/x)$ of $(\exists x)P$ provided (1) the individuating constant a does <i>not</i> occur in <i>any</i> premise or in an active proof (or subproof) <i>prior</i> to its arbitrary introduction in the assumption $P(a/x)$ and (2) the individuating constant a does not occur in proposition Q discharged from the subproof.</p> | $ \begin{array}{l} (\exists x)P \\ \left \begin{array}{l} P(a/x) \\ \cdot \\ \cdot \\ \cdot \\ Q \end{array} \right. \\ Q \end{array} $ | $\exists E$ |
|---|--|-------------|

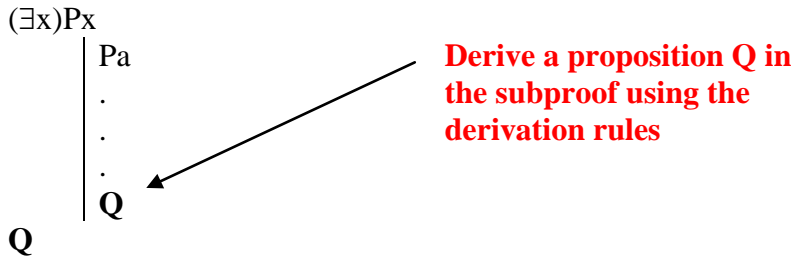
Let's focus on the first part of this rule. Later on, we will look at the second part. What $\exists E$ says is that *starting* from an existentially quantified proposition, e.g. $(\exists x)Px$, you can infer some other proposition Q .



But, you cannot reason directly from $(\exists x)Px$ to Q . You will have to go through a couple intermediate steps to get there. Let's look at those steps. First, all uses of $\exists E$ require you to make an assumption, but not any assumption, an assumption where you remove the existential quantifier and replace bound variables with individual constants (names).



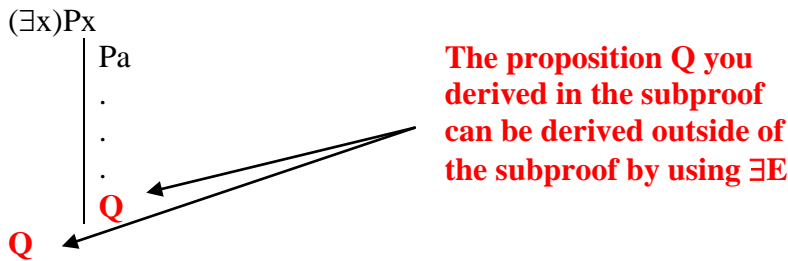
Next, in that subproof, you will need to derive a proposition Q .



Let's consider a real example of the above:

| | | |
|---|---------------------------|---------------|
| 1 | $(\exists x)Px$ | P |
| 2 | Pa | A |
| 3 | $Pa \vee Ra$ | $2 \vee I$ |
| 4 | $(\exists x)(Px \vee Rx)$ | $3 \exists I$ |
| 5 | Q | |

Now that you have derived a proposition **Q** within the subproof, you can rewrite that proposition outside of the subproof using $\exists E$.



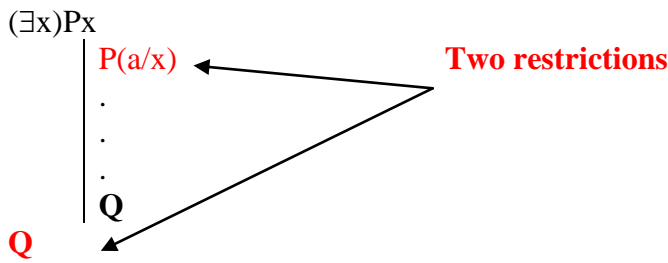
Continuing our example above:

| | | |
|---|---------------------------|--------------------|
| 1 | $(\exists x)Px$ | P |
| 2 | Pa | A |
| 3 | $Pa \vee Ra$ | $2 \vee I$ |
| 4 | $(\exists x)(Px \vee Rx)$ | $3 \exists I$ |
| 5 | $(\exists x)(Px \vee Rx)$ | $1, 2-4 \exists E$ |

So, $\exists E$ allows us to derive a proposition **Q** from an existentially quantified proposition $(\exists x)P$. However, there are some restrictions on the use of $\exists E$. But what about the rest of the rule? Let's turn to the second half of $\exists E$ now.

| | | |
|--|--|-------------|
| <p>Existential Elimination ($\exists E$) From an existentially quantified expression $(\exists x)P$, an expression Q can be derived from the derivation of an assumed substitution instance $P(a/x)$ of $(\exists x)P$ provided (1) the individuating constant a does not occur in any premise or in an active proof (or subproof) prior to its arbitrary introduction in the assumption $P(a/x)$ and (2) the individuating constant a does not occur in proposition Q discharged from the subproof.</p> | $ \begin{array}{l} (\exists x)P \\ \left \begin{array}{l} P(a/x) \\ \cdot \\ \cdot \\ \cdot \\ Q \end{array} \right. \\ Q \end{array} $ | $\exists E$ |
|--|--|-------------|

The second half of $\exists E$ places important restrictions on what can be assumed and the proposition derived outside of the subproof.



The first restriction states that if we want to use $\exists E$, we cannot assume a proposition that involves an individual constant ($a - v$) already in the proof. For example

| | | | |
|---|----------------------|---------------------|--|
| 1 | $(\exists z)Wzz$ | P | |
| 2 | $Wbb \rightarrow Lc$ | P | |
| 3 | Wbb | A | ← NO! b is already in the proof |
| 4 | Lc | $2,3 \rightarrow E$ | |
| 5 | Lc | $1,3-4 \exists E$ | |

The second restriction states that if you assume a proposition that involves an individual constant, you cannot derive a proposition that has *that* individual constant outside of the proof.

| | | | |
|---|------------------|--------------------------|--------------------------------------|
| 1 | $(\exists z)Wzz$ | P | |
| 2 | Wbb | A / $\exists E$ | ← NO! b was assumed at line 2 |
| 3 | $(\exists x)Wbx$ | $2 \exists I$ | |
| 4 | $(\exists x)Wbx$ | $1, 2-3 \exists E$ — NO! | |